Design of Functionally Graded Piezocomposite Materials Using Topology Optimization with Polygonal Mesh

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Acknowledgements:
Outline

- Introduction and Motivation
- Objective
- Theoretical Topics
  - Polygonal Mesh
  - Topology Optimization
- Numerical Results
- Conclusions and Future Works
Piezoelectric Materials

**Constitutive Equations:**

\[
[T] = [c^E][S] - \{e\}^t\{E\} \\
{D} = [\varepsilon^S]{E} + [e][S]
\]

- \([T]\) – stress tensor
- \({E}\) – electric field vector
- \([S]\) – strain tensor
- \({D}\) – electric displacement vector
- \([c^E]\) – elastic tensor obtained at constant electric field
- \([e]\) – piezoelectric tensor
- \([\varepsilon^S]\) – dielectric tensor obtained at constant strain

**Examples:**
- Quartz (natural)
- Ceramic (PZT5A, PMN)
- Polymer (PVDF)

**Applications:** ultrasonic transducers, actuators, pressure sensors, accelerometers, sonar, hydrophones, MEMS, etc...
Motivation

**Motivation:** to design materials used in piezoelectric sensors

**Examples:**

- Accelerometer
- Pressure sensor

**Main objective**

- Maximize the output voltage
- Need to reduce their stiffness

- Piezoelectric materials are too stiff
Piezocomposite Materials

- Combines piezoelectric material → non-piezoelectric materials
- Better performance than pure materials
- Depends on: volume fractions, material properties, shape of inclusions
- Interfaces: might present stress concentrations, which may cause material fracture and fatigue.

Functionally Graded Materials (FGM)

FGMs possess continuously graded properties with gradual change in microstructure which avoids interface problems, such as, stress concentrations.


**Microstructure**

- **Ceramic Phase**
  - Ceramic matrix with metallic inclusions
- **Transition region**
- **Metallic matrix with ceramic inclusions**
- **Metallic Phase**

**Example**: Cu-Ni FGM disk

- **Top View**
  - Ni
- **Front View**
- **Bottom View**
  - Cu
Homogenization Method

→ Calculation of the effective properties

→ The homogenized properties depend on the volume fractions of constituent materials, its properties, and shape of inclusions in the unit cell.

Topology Optimization Method (TOM)

→ Design of piezocomposites FGM

Piezocomposite Design Using TOM

1. Design domain
2. Discrete domain
3. Optimum topology
4. Verification
5. Post-processing
6. Manufacturing

(unit cell)
Previous works

Material Design

- Distribution of material phases in a unit cell that optimizes the properties of a composite (Cherkaev, Kohn, 1997)
- Design of materials with prescribed parameters:
  - elastic materials (Sigmund, 1995)
  - piezoelectric materials (Silva et al, 1999)
  - thermoelastic materials (Torquato et al, 2003)

Stress Calculation In Unit Cells

- Preprocessing And Postprocessing For Materials Based On The Homogenization Method With Adaptive Finite-Element Methods (Guedes and Kikuchi, 1990)
- Determination of the micro stress field in composite by homogenization method (Ni et al, 2006)

FGM Material Design

- Optimal design of FGM composites with prescribed properties (Paulino et al, 2008)
To design piezocomposite materials based on FGM concept using topology optimization and homogenization methods, in order to maximize the output voltage of piezoelectric sensors.
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Polygonal Mesh

**Generation:**

a) Populate the domain with a desired number of ‘seeds’

b) Calculate auxiliary points in the boundary domain

c) Construct the Voronoi diagram

**Example:**

- regular square mesh
- one-node connections
- polygonal mesh

Topology Optimization Method

→ How to change the material from zero to one?

\[ \Gamma^H = \rho^p \Gamma_A + \left(1 - \rho^p\right) \Gamma_B \]

\( \rho \): design variable for material distribution

\( \Gamma \): material properties
- \( c \) – elastic properties
- \( e \) – piezoelectric properties
- \( \varepsilon \) – dielectric properties

→ Classical concept of orientation optimization in a finite element

\( \theta_i \): design variable for angle

Common problem: large risk of obtaining a local optimum solution
Topography Optimization Method

→ Material Model: Discrete Material Optimization (DMO)

\[ \Gamma_0 = \sum_{i=1}^{n_c} w_i \Gamma_i = w_1 \Gamma_1 + w_2 \Gamma_2 + \cdots + w_{n_c} \Gamma_{n_c} \]

Material properties

Design variables

\[ w_i = \frac{\hat{\psi}_i}{\sum_{k=1}^{n} \hat{\psi}_k}, \text{ where } \hat{\psi}_i = (\rho_i)^p \prod_{j=1, j \neq i}^{n_c} \left[ 1 - (\rho_i)^p \right] \]


How to control the gradation?

Projection Functions


Topology Optimization Method

**CAMD: Continuous Approximation of Material Distribution**

- Design variable
- Displacements (x and y) and electrical potential

\[
\rho(x) = \sum_{i=1}^{\text{nnodes}} \rho_i N_i(x)
\]


**Grade Finite Element Concept**

\[
D^e(x, y) = \sum_{i=1}^{4} D^e_i N_i
\]

Topology Optimization Method

How to measure it?

Electromechanical Coupling Coefficient:
\[ d = \frac{(c_{33} - c_{13})e_{13}}{c_{11}c_{33} - c_{13}^2} \]

\( c \): elastic coefficients
\( e \): piezoelectric coefficients

How to measure it?

electrical energy \rightarrow mechanical energy

forces

electrodes

Formulation of the Optimization Problem:

Maximize:
\[ F(\rho, d) \]

subject to:
\[ 0 \leq \rho \leq 1 \]

symmetry conditions
gradation control

Solver:

Method of Moving Asymptotes (MMA)

Topology Optimization Method

Flowchart:

1. Data Input
2. Random material distribution
3. Calculate homogenized properties
4. Calculate objective function
5. Converged? (yes/no)
   - yes: Results Output
   - no: Calculate sensitivities
     - Solve optimization problem
     - Update design variables
     - Apply gradation control
Numerical Results: Parameters

**Materials:**
- PZT-5A ceramic
- Epoxy polymer

**Mesh:** 60 elements

**Initial guess:** random

**Boundary Conditions:**
- material distribution is symmetric in x and y
- polarization direction is symmetric in y

**Example:**
- \( r_{\text{grad}} = 2.5\% \)

- PZT-5A
- Epoxy

- design domain
- unit cell
- periodic matrix

(homogenized properties considering full unit cell)
Numerical Results

**Objective Function Curve**

- Objective Function: $F$ vs. iterations
- Random initial conditions (random)
Numerical Results: Optimized Unit Cells

<table>
<thead>
<tr>
<th>$r_{\text{grad}}$</th>
<th>Unit Cell</th>
<th>Periodic Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td><img src="PZT-5A_Unit_Cell_2.5%25" alt="Image" /></td>
<td><img src="PZT-5A_Periodic_Matrix_2.5%25" alt="Image" /></td>
</tr>
<tr>
<td>5%</td>
<td><img src="PZT-5A_Unit_Cell_5%25" alt="Image" /></td>
<td><img src="PZT-5A_Periodic_Matrix_5%25" alt="Image" /></td>
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<tr>
<td>7.5%</td>
<td><img src="PZT-5A_Unit_Cell_7.5%25" alt="Image" /></td>
<td><img src="PZT-5A_Periodic_Matrix_7.5%25" alt="Image" /></td>
</tr>
<tr>
<td>10%</td>
<td><img src="PZT-5A_Unit_Cell_10%25" alt="Image" /></td>
<td><img src="PZT-5A_Periodic_Matrix_10%25" alt="Image" /></td>
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<tr>
<td>12.5%</td>
<td><img src="PZT-5A_Unit_Cell_12.5%25" alt="Image" /></td>
<td><img src="PZT-5A_Periodic_Matrix_12.5%25" alt="Image" /></td>
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Numerical Results: Optimized Unit Cells

\[ r_{\text{grad}} = 2.5\% \quad r_{\text{grad}} = 5\% \quad r_{\text{grad}} = 7.5\% \quad r_{\text{grad}} = 10\% \quad r_{\text{grad}} = 12.5\% \]

PZT-5A

Unit Cell

Periodic Matrix

Epoxy
Numerical Results: Optimized Unit Cells

- \( r_{\text{grad}} = 2.5\% \)
- \( r_{\text{grad}} = 5\% \)
- \( r_{\text{grad}} = 7.5\% \)
- \( r_{\text{grad}} = 10\% \)
- \( r_{\text{grad}} = 12.5\% \)

**Unit Cell**

- PZT-5A
- Epoxy

**Periodic Matrix**
Numerical Results: Optimized Unit Cells

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- PZT-5A
- Epoxy
Numerical Results: Optimized Unit Cells

- $r_{\text{grad}} = 2.5\%$
- $r_{\text{grad}} = 5\%$
- $r_{\text{grad}} = 7.5\%$
- $r_{\text{grad}} = 10\%$
- $r_{\text{grad}} = 12.5\%$

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<td></td>
<td><img src="image_url" alt="Diagram" /></td>
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Numerical Results

\( r_{\text{grad}} = 2.5\% \)

\( r_{\text{grad}} = 5.0\% \)

\( r_{\text{grad}} = 7.5\% \)

\( r_{\text{grad}} = 10.0\% \)

\( r_{\text{grad}} = 12.5\% \)

Eletromechanical Coupling Coefficient

- optimized cells
- PZT-5A

\[ \text{d (pC/N^2)} \]

\[ r_{\text{grad}}(\%) \]
Numerical Results

Output voltage signal

Microscopic Stress

Micro Stress Graphics

Output Voltage Signal (V/V₀)

optimized cells

PZT-5A

Gradation Control (%)
**Numerical Results**

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<th>Microscopic Stress</th>
<th>Micro Stress Graphics</th>
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<tbody>
<tr>
<td><strong>unit cell</strong></td>
<td><strong>$\sigma/\sigma_0$</strong></td>
<td></td>
</tr>
<tr>
<td>$\Gamma_{grad} = 2.5%$</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>$\Gamma_{grad} = 5%$</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>$\Gamma_{grad} = 7.5%$</td>
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<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>$\Gamma_{grad} = 10%$</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td>$\Gamma_{grad} = 12.5%$</td>
<td><img src="image9.png" alt="Image" /></td>
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Numerical Results

Output voltage signal  Microscopic Stress  Micro Stress Graphics

Maximum Stress ($\sigma/\sigma_0$)

- **optimized cells**
- **non-composite material**

Maximum Gradient Stress

- **optimized cells**
- **non-composite material**
Conclusions

FGM concept

• decreases the objective function values
• reduces maximum microscopic stress
• reduces microscopic stress concentrations

DMO material model

• The variation of the polarization directions inside the unit cell helps to increase the objective function.

TRADE-OFF
The End

Thank you!

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