Challenges and Advances in System Reliability-Based Optimization of Structural Topology

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• **System Reliability** Based Design Optimization (SRBDO)
• **Multiresolution** Topology Optimization (MTOP)
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Reliability Based Design Optimization

>> Deterministic Optimization

$$\min_{d, \mu_X} \ f(d, \mu_X)$$

subject to

$$g_i(d, X) > 0$$

$$d^L \leq d \leq d^U$$,  $$\mu_X^L \leq \mu_X \leq \mu_X^U$$

>> Reliability-Based Design Optimization (RBDO)

$$\min_{d, \mu_X} \ f(d, \mu_X)$$

subject to

$$P_{sys} = P(E_{sys}) = P\left[ \bigcup_{k \in C_x} \bigcap_{i \in C_x} g_i(d, X) \leq 0 \right] \leq P_{sys}^i$$

Low probability of failure

High probability of failure

Safe

Unsafe

Objective function increase $$f(d, \mu_X)$$
System Reliability Based Design Optimization

- Component RBDO

\[
\min_{d, \mu_X} f(d, \mu_X) \\
\text{s.t.} \quad P[g_i(d, X) \leq 0] \leq P^i_i \quad i=1,\ldots,n \\
d^L \leq d \leq d^U, \quad \mu_X^L \leq \mu_X \leq \mu_X^U
\]
Matrix-based System Reliability Method

- **Convenient**: matrix-based procedures for \( c \) and \( p \); easy SRA calculation (inner product)
- **General**: uniform application to series, parallel, and any general systems
- **Flexible**: inequality-type information; incomplete information (“LP bounds” method)
- **Efficient**: no need to re-compute \( p \); replace \( c \) for SRA of a new event
- **Common Source Effect**: can account for statistical dependence between components
- **Decision Support**: parameter sensitivities, component importance measure; inferences
Proposed Approach: SRBDO using MSR

- Adopts a single-loop RBDO (Liang et al. 2007)
  - Use single-loop performance measure approach (PMA) using Karush-Kuhn-Tucker (KKT) condition
- Use MSR method to compute $P_{sys}$ and its gradients

\[
\begin{align*}
\min_{d, \mu_X, P'_1, \ldots, P'_n} & \quad f(d, \mu_X) \\
\text{s.t.} & \quad g_i(d, X(U'_i)) \geq 0 \quad i=1, \ldots, n \\
& \quad P_{sys} = \begin{cases} 
\int_s c^T p(s) f_s(s) ds \leq P^t_{sys} & \text{dependent} \\
\quad c^T p \leq P^t_{sys} & \text{independent}
\end{cases}
\end{align*}
\]
Proposed Approach: SRBDO using MSR (contd.)

• Sensitivity w.r.t. design variables $\theta = \{d, \mu_x\}$

$$\frac{\partial P_{sys}}{\partial \theta} = \int \mathbf{c}^T \frac{\partial \mathbf{p}(s)}{\partial \theta} f_s(s) ds$$

$$\frac{\partial \mathbf{p}(s)}{\partial \theta} = \left[ \mathbf{p}(s)^{(1)} \mathbf{p}(s)^{(2)} \cdots \mathbf{p}(s)^{(n)} \right] \frac{\partial \mathbf{P}(s)}{\partial \theta} = \hat{\mathbf{P}}(s) \frac{\partial \mathbf{P}(s)}{\partial \theta}$$

$$\mathbf{P}(s) = [P_1(s) P_2(s) \cdots P_n(s)]^T$$

→ Use probabilities and sensitivities by component reliability analysis (FORM)

• Sensitivity w.r.t. component failure probability $P_i^t$

$$\frac{\partial P_i(s)}{\partial P_i} = \frac{\partial P_i(s)}{\partial \beta_i} \cdot \frac{\partial \beta_i}{\partial P_i} = -\frac{\partial P_i(s)}{\partial \beta_i} \cdot \frac{1}{\phi(-\beta_i)}$$
SRBDO of Truss System


\[
\min_{\mathbf{d}=\{A_1,\ldots,A_6\}} f(\mathbf{d}) = \sqrt{2}(A_1 + A_2) + A_3 + A_4 + A_5 + A_6
\]

\[s.t.\quad P_{sys} = P \left[ \bigcup_{k=1}^{15} \bigcap_{i \in C_k} g_i(\mathbf{d}, \mathbf{X}) \leq 0 \right] \leq P_{sys}^t = 0.001\]

\[g_i(\mathbf{d}, \mathbf{X}) = A_i F_i - 0.707 F_A \quad i = 1, 2\]
\[A_i F_i - 0.500 F_A \quad i = 3, \ldots, 6\]

\[A_1, A_2, A_3, A_4, A_5, A_6 \geq 0\]

→ Minimize total weight of the system

→ Definition of system failure: at least two members fail (cut-set systems): effects of load re-distributions NOT considered

Random Variables
(Gaussian distribution)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member strength $F_i$, $i=1,..6$ (Mpa)</td>
<td>745</td>
<td>62</td>
</tr>
<tr>
<td>Applied load $F_A$ (kN)</td>
<td>4450</td>
<td>45</td>
</tr>
</tbody>
</table>
### SRBDO of Truss System (contd.)

<table>
<thead>
<tr>
<th>Members</th>
<th>Area: $A_i \times 10^{-3}$ mm$^2$</th>
<th>Reliability Index: $\beta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MacDonald &amp; Mahadevan</td>
<td>SRBDO/MSR</td>
</tr>
<tr>
<td>1</td>
<td>18.43</td>
<td>17.89</td>
</tr>
<tr>
<td>2</td>
<td>18.27</td>
<td>17.89</td>
</tr>
<tr>
<td>3</td>
<td>13.51</td>
<td>13.20</td>
</tr>
<tr>
<td>4</td>
<td>13.44</td>
<td>13.20</td>
</tr>
<tr>
<td>5</td>
<td>13.33</td>
<td>13.20</td>
</tr>
<tr>
<td>6</td>
<td>13.09</td>
<td>13.20</td>
</tr>
</tbody>
</table>

\[ f(x) \quad 105.24 \quad > \quad 103.36 \]

- Better optimal design (i.e. less total weight) and symmetric results
- Monte Carlo simulations (c.o.v. = 0.03, $10^6$ times) on the system failure probability: $P_{sys} = 0.00107$ (cf. MSR gives 0.001)
SRBDO of Truss System (contd.)

- **Conditional probability Importance Measure (CIM)**

\[ \text{CIM}_i = P(E_i | E_{\text{sys}}) = \frac{P(E_i E_{\text{sys}})}{P(E_{\text{sys}})} = \frac{c_i^T p}{c^T p} \]

- Relative contribution of components to the system failure probability (can be computed efficiently by MSR method)
SRBDO of Truss System (contd.)


- Effects of load re-distributions (sequential failures)
- Effects of correlation between random variables and between components
### Topology Optimization

- **Structural design** optimization: the optimal sizes or shapes for a given layout and connectivity

![Bridge Structure](image)

- **Topology** optimization: the best topology, shape, size from a domain and boundary conditions

![Element Removal](image)
High Resolution Topology Optimization

- Large-scale (high resolution) TOP
  - Large number of finite elements
  - Computationally expensive

- Existing approaches for high resolution TOP
  - Parallel computing (Borrvall and Petersson, 2000)
  - Fast solvers (Wang et al. 2007)
  - Approximate reanalysis (Amir et al. 2009)
  - Adaptive mesh refinement (de Stuler et al. 2008)
Multiresolution Topology Optimization (MTOP)

- Conventional element-based approach (Q4/U)
- Proposed MTOP approach (Q4/n25)
TO of **2D** Cantilever Beam

Configuration

MTOP Q4/n25 FE mesh 48x16

Q4/U FE mesh 240x80

Q4/U FE mesh 48x16
3D Bridge

Configuration

MTOP B8/n125
36,000 elements

Existing design
System Reliability-Based Topology Optimization

- **Discrete structures**
  - Mogami et al. (2006)
  - Truss examples

- **Continuum structures**
  - Silvia et al. (2010)
  - Assumes component events (limit-states) are statistically independent
SRBTO of 3D Cube

- **Objective**: minimize volume $V(\rho)$
- **Limit-states**: $g_i(\rho, \bar{F}_i) = 120 - C_i(\rho, \bar{F}_i)$, $i = 1, 2$
- **Random loads**: $\mathbf{F} \sim (F_1, F_2, F_3) \sim N(100,10), N(0,30), N(0,40)$
- **Load cases**: $\bar{F}_1 = (F_1, F_2)$, $\bar{F}_2 = (F_1, F_3)$

- **Constraints**:
  - **Deterministic TO (DTO)**: $g_i(\rho, \mathbf{f}) > 0$, $i = 1, 2$
  - **Component RBTO (CRBTO)**: $P(g_i(\rho, \bar{F}_i) \leq 0) \leq P_i^\prime$, $i = 1, 2$
  - **System RBTO (SRBTO)**: $P(\bigcap g_i(\rho, \bar{F}_i) \leq 0) \leq P_{sys}^\prime$
Optimal Topologies

- DTO: $\text{volfrac} = 6.3\%$
- CRBTO: $\text{volfrac} = 24.4\%$
- SRBTO: $\text{volfrac} = 22.3\%$
[Ongoing] Improve Accuracy by Second Order Reliability Method

- FORM-based SRBTO
- SORM-based SRBTO

- constraint on $P_{sys}$
- MCS on SORM-based
- MCS on FORM-based

![Graph showing the relationship between optimal volume fraction and standard deviation for FORM-based SRBTO and SORM-based SRBTO.]

![Graph showing the relationship between system probability and standard deviation for constraint on $P_{sys}$ and MCS on SORM-based and FORM-based.]

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[Ongoing] SRBTO of 3D Building
Summary & Conclusions

• **System Reliability** based Design Optimization (SRBDO/MSR)
  i. Enables uniform applications to **general system events**
  ii. Accounts for **statistical dependence**
  iii. Helps utilize **gradient-based optimizer**
  iv. Effect of **load re-distributions** (truss system example)

• **High Resolution** topology optimization (MTOP)
  i. High resolution with **lower computational cost**
  ii. **2D beam and 3D bridge** examples

• **SRBTO with high resolution**
  i. SRBDO/MSR + MTOP
  ii. **3D cube** example and ongoing applications to **3D building core**
Thank You!