A Bar and Hinge Model for Scalable Structural Analysis of Origami

Evgueni T. Filipov¹, Glaucio H. Paulino¹,², Tomohiro Tachi³

Motivation

Origami is popular in science and engineering because of:
- self-assembly • deployment • compact stowage • adaptability

Applications can range in scale from metamaterials and micro-robotics to aerospace systems and deployable architecture.

In this research we improve a bar and hinge model to enable scalable, efficient and simplified structural analysis of origami.

Elastic Behaviors of Origami

Stretching and Shear of Panels

An indeterminate frame of bar elements is used to capture in-plane isotropic behavior of the panels.

Young's modulus: E

Thickness: t

Poisson's ratio: ν

Bar stiffness definition: \( A_{x/y} = t \frac{W^2 - w^2}{2W(1-\nu)} \)

Panel stiffness scaling: \( E_x - k \frac{L_x}{t} \), the bending modulus is:

\( k = E_t \frac{E_t^2}{12(1-\nu^2)} \)

Bending of Panels

Out-of-plane bending curvature localizes along the shorter diagonal of the panels. Rotational hinges simulate the stiffness.

Bending of Fold Lines

Bending curvature is concentrated on the fold. Ratio \( K_{FP} \) relates bending stiffness of fold to panel, and depends on material and physical properties.

Assembly of Bar and Hinge Model

- Length scale dependent
- Isotropic
- Includes material properties \( E, t, ν \)

Eigenvalue Analyses

\[ K = \text{Stiffness matrix} \quad K = \lambda_i M \quad i = 1, \ldots, N_{\text{def}} \]

Analysis of single tube

- Eigenvalue analyses can be used to predict the kinematic motion of the structure
- Increasing the band-gap makes a stiff, yet deployable origami
- The model approximates modes with panel and fold bending well, but underestimates the stiffness of stretching and shearing

Static Stiffness Analyses

\[ F = K \Delta \]

Load applied perpendicular to X axis

Bar and hinge model

Detailed FE model

- The bar and hinge model captures the anisotropy for reconfiguring polygonal tubes, but overestimates stiffness

Model Accuracy and Efficiency

Convergence

- The efficient bar and hinge model is provides an accuracy comparable to detailed FE models for a same number of degrees of freedom

References


Benefits and Extensions of Model

- Model is simple to understand, implement, modify and use. This makes it valuable to the growing community of origami researchers and enthusiasts
- The bar and hinge model is scalable, isotropic and incorporates material properties \( t, E, ν \)
- It provides sufficient accuracy for global structural analysis of origami

- The model is efficient in comparison to detailed FE analyses making it suitable for extensions such as:
  - Large displacement simulations
  - Parametric variations for geometric design
  - Optimization of cellular origami type structures

Limitations

- The bar and hinge model cannot capture localized effects accurately
- Stiffness for stretching and shearing of the panels is overestimated in comparison to the bending deformations
- The factor relating fold to panel stiffness \( (K_{FP}) \) and the factor defining the panel bending \( (C_y) \) stiffness have not been thoroughly investigated

Acknowledgements: