A Paradigm for Higher Order Mixed Polygonal Elements for Finite Elasticity

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**Motivation**

- Soft organic materials, such as electro- and magneto-active elastomers and gels, hold tremendous potential for new high-end technologies, e.g., next generation sensors and actuators.
- Soft materials often possess complex microstructures, with underlying local deformations which are typically larger than macroscopic ones.
- The gradient correction leads to optimal convergence for linear and quadratic polygonal elements.
- Polygonal elements have advantages in modeling inclusions with arbitrary geometries, as well as incorporating periodic boundaries.
- Polygonal elements (both linear and quadratic) appear to be more tolerant to large local deformations than classical finite elements.

**Formulation and Polygonal FEM Approximations**

- Find \((u^*, p^*)\) such that:
  \[
  \Pi(u^*, p^*) = \min_u \max_p \Pi(u, p)
  \]
- \(\Pi(u, p) = \int_{\Omega_0} -W(X,F(u),p) + p \det(F(u)) \, d\Omega_0 - \int_{\Omega_0} f_0 \cdot u \, d\Omega_0 - \int_{\partial\Omega_0} t_0 \cdot u \, d\partial\Omega_0\)
- Polygonal shape functions:
  - Linear (Floater, 2003)
  - Quadratic (Rand et al., 2014)
- Displacement and pressure approximations:
  - \(\Phi, \Phi_h\) : quadrature of order \(2k - 2\) over element \(E\)
  - \(\Phi_e\) : exact integral over element \(E\)
  - \(\Phi_h\) : quadrature of order \(2k - 1\) over the boundary of \(E\)
  - \(\mathcal{M}_k(E)\) : element local space polynomials of order \(k\)
- The correction to the gradient is defined as:
  - For any given \(\nabla u \in \mathcal{M}_k(E)\), the corrected gradient \(\nabla_{\text{corr}} u\) is the one satisfies:
  - \(\nabla_{\text{corr}} u = \nabla u - \nabla (\nabla u)\)
  - \(\int_E p \cdot \nabla u = \int_{\Omega_0} (p \cdot N) \, d\Omega_0 - \int_{\partial\Omega_0} p \cdot n_0 \, d\partial\Omega_0\)

**Conclusion**

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**Reference**