Introduction

Sizing Optimization

Shape Optimization

Topology Optimization

Extensions to multiscale, multiphysics problems:

Negative Poisson’s Ratio

Negative thermal expansion

Courtesy of Prof. John Halloran, Material Science and Engineering, University of Michigan

Motivation

Uniform grids are traditionally used to parameterize and analyze design

In addition to numerical instabilities, the constrained geometry of these meshes can bias the orientation of members in optimal design

This work examines the use of polygonal finite element in topology optimization to address these issues

Polygonal Mesh Generation

The use of auxiliary points guarantee that the resulting Voronoi diagram includes an approximation to the boundary

The domain is described by the zero level set of a given function:

\[ f(x) = 0 \]

Placement and reflection of seeds can be carried out generically using a signed distance function:

\[ x_R = x - 2d(x) \frac{\nabla d(x)}{|\nabla d(x)|} \]

The resulting mesh using the Centroidal Voronoi Tessellation of the point set:

Finite element formulation

For a convex polygon, the Laplace interpolant is defined as:

\[ \phi_i(x) = \frac{w_i(x)}{\sum_{j=1}^{n} w_j(x)} \]

where \( w_i(x) = \frac{s_i(x)}{h_i(x)} \)

An isoparametric mapping from regular \( n \)-gons to any convex polygon is constructed using these shape functions

Weak form integrals are evaluated by triangulating the parent element and using the usual quadrature rules

Numerical performance

Cook’s problem consisting of a tapered panel subjected to uniform shear loading:

Polygonal elements are not as stiff as the quad elements

Mesher used: note the progressive refinement for quads and independent refinement for polygons

Minimum compliance design:

The T6 mesh suffers from the limitation of its geometry while the CVT meshes have the flexibility to represent the optimal layout:

Conclusions

Solutions of discrete topology optimization problems with fixed mesh representation include a form of mesh dependency that stems from the geometric features of the spatial discretization

To address this problem, we employ fully unstructured meshes to reduce the influence of simple geometry on optimization solutions