Simulation of Crack Propagation in Asphalt Concrete Using an Intrinsic Cohesive Zone Model

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Abstract: This is a practical paper which consists of investigating fracture behavior in asphalt concrete using an intrinsic cohesive zone model (CZM). The separation and traction response along the cohesive zone ahead of a crack tip is governed by an exponential cohesive law specifically tailored to describe cracking in asphalt pavement materials by means of softening associated with the cohesive law. Finite-element implementation of the CZM is accomplished by means of a user subroutine using the user element capability of the ABAQUS software, which is verified by simulation of the double cantilever beam test and by comparison with closed-form solutions. The cohesive parameters of finite material strength and cohesive fracture energy are calibrated in conjunction with the single-edge notched beam (SE(B)) test. The CZM is then extended to simulate mixed-mode crack propagation in the SE(B) test. Cohesive elements are inserted over an area to allow cracks to propagate in any direction. It is shown that the simulated crack trajectory compares favorably with that of experimental results.

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Introduction

Cracking has occurred in nearly all types of asphalt overlays due to mechanical and environmental loadings. Because cracking causes water penetration, thereby weakening the foundation of the pavement structure and contributing to increased roughness, a number of studies have been conducted to obtain a better understanding of cracking mechanisms and to tackle the cracking problem in asphalt concrete. Majidzadeh et al. (1971) made an early attempt to study crack propagation using fracture testing. Abdulshafi and Majidzadh (1985) applied the J-integral concept to fatigue and fracture of asphalt mixtures in conjunction with a disk-shaped specimen. Kim and El Hussein (1995) used three-point bending tests to explore fracture behavior of asphalt concrete and to evaluate fracture toughness of asphalt concrete at low temperatures. Jacobs et al. (1996) employed Paris’ law to analyze cracking in asphalt concrete and to obtain more insight into the crack propagation and resistance of asphalt mixes. Bhurke et al. (1997) developed a test protocol to calculate fracture toughness of asphalt concrete at low temperatures. Castell et al. (2000) investigated fatigue crack growth in a laboratory beam specimen and layered pavements using the code Franc2D/L. Several other researchers (Owusu-Antwi et al. 1998; Shen and Kirkrner 1999; Sangpetngam et al. 2004) have applied fracture mechanics principles in the study of cracking in asphalt concrete laboratory specimens and pavements. However, most studies of fracture of asphalt concrete have been limited either to experimental investigation or to the analysis of stationary cracks. In this work, a powerful numerical scheme using the cohesive zone model (CZM) concept is introduced to investigate the fracture behavior of asphalt concrete and to simulate crack initiation and propagation of both mode I and mixed-mode cracks.

Cohesive zone models have been used to simulate fracture of both homogeneous and nonhomogeneous materials. Barenblatt (1959, 1962) proposed an early cohesive model to study brittle fracture, and Dugdale (1960) adopted a process zone concept to investigate materials exhibiting plasticity. Xu and Needleman (1994) presented an intrinsic potential-based model where cohesive elements are inserted along either lines or regions in advance, and they implemented this model by means of the finite-element method. Despite great success in simulating crack propagation, this type of cohesive model introduces artificial compliance due to the initial prepeak slope of the intrinsic cohesive law. To alleviate such problems, Espinosa and Zavattieri (2003) used a bilinear model to reduce the compliance by adjusting the initial slope of the cohesive law. An alternative cohesive law was proposed by Camacho and Ortiz (1996). They presented a stress-based extrinsic cohesive law where a new surface was previously bonded. Subsequent investigations were carried out to apply the cohesive fracture modeling in several areas, such as concrete (Mosalam and Paulino 1997; Ruiz et al. 2001), dynamic crack growth (Sieg mund and Needleman 1997; Ruiz et al. 2001), viscoelasticity (Rahulkumar et al. 2000).
Cohesive Zone Model

The CZM concept is provided herein, and its formulation using the principle of virtual work is presented. The force vector and tangent matrix, which are variables to be defined in the UEL, are formulated. Finally, the potential-based effective CZM is introduced.

CZM Concept

The cohesive zone model provides a computationally efficient way to simulate damage occurring in a process zone located ahead of a crack tip (see Fig. 1). This approach, which involves nonlinear constitutive laws described by the displacement jump and the corresponding traction along the interfaces, provides a phenomenological model to simulate fracture behavior such as crack nucleation, initiation, and propagation.

Fig. 1 illustrates the CZM concept in the opening mode (pure mode I) where $T_n$ and $\delta_n$ denote normal traction and normal displacement jump, respectively. The material crack tip indicates a point where traction is zero and the cohesive zone tip is a point where the traction reaches a maximum. The cohesive zone is defined as the region between the material crack tip and the cohesive zone tip where complicated fracture behaviors, including inelasticity, occur. The cohesive surfaces are joined together by a cohesive traction, which depends upon the displacement jump across crack faces. As the displacement jump increases due to an increase in external force or compliance in a structure, the traction first increases, then reaches a maximum, and finally decays monotonically to zero. The separation-material response depends on the material strength, critical displacement, and cohesive fracture energy, which represent the cohesive parameters.

CZM Formulation

The principle of virtual work considering the cohesive element contribution is given as

$$\int_V \sigma : D^* dV - \int_S T : \delta^* dS - \int_S T : u^* dS = 0 \quad (1)$$

where $\sigma =$ Cauchy stress tensor; $D^*$ = virtual strain tensor; $\sigma : D^*$ indicates $\sigma_{ij}D_{ij}^*$; $T=$ traction on the boundary; $\delta^* =$ virtual displacement jump across the cohesive surface; $u^* =$ virtual displacement in the bulk (background) material; and $S$ and $V =$ current (deformed) surface and volume, respectively.

An implicit displacement-based finite-element scheme requires evaluation of several terms, such as a force vector and a tangent matrix based on different numerical schemes. Evaluation of the tangent matrix and the force vector is necessary for the iteration of the Newton-Raphson method and the Riks method, and an additional term, which is an incremental load vector, needs to be defined for the iteration of the Riks method (ABAQUS 2002). The force vector and the tangent matrix of the cohesive elements are obtained from the second term of Eq. (1) and the first variation of the second term in Eq. (1), respectively. Notice that the incremental load vector of the cohesive elements to be defined for the Riks method is zero for the CZM, because a cohesive force is an internal force and, as a result, is independent of the Riks load parameter.

The virtual cohesive element work is given as

$$W_{coh}^* = \int_S (T_n \delta_n^* + T_s \delta_s^*) dS \quad (2)$$

where subscript “coh” indicates cohesive; and $T_n$ and $T_s$ are normal and shear tractions, respectively. Moreover, $\delta_n^*$ and $\delta_s^*$ are virtual normal and shear displacement, respectively, and can be expressed in terms of the shape functions and virtual nodal displacement jump along the normal and shear direction given by

$$\delta_n^* = N \delta_n, \quad \delta_s^* = N \delta_s,$$  

where $N =$ shape functions relating quantities at nodal points to those at Gauss points; and $\delta_n$ and $\delta_s =$ virtual normal and shear
The parameter \( \beta \), which is defined as the ratio between the maximum normal traction and the shear traction, is introduced to express the formulation with single effective displacement by assigning different weights for displacements and tractions along the normal and shear directions. In this simulation, \( \beta = \sqrt{2} \) (Camacho and Ortiz 1996).

The exponential form for the free energy potential is given (Xu and Needleman 1994) by

\[
\phi = \epsilon \sigma, \delta_{n} \left[ 1 - \left( 1 + \frac{\delta_{s}}{\delta_{c}} \right) \exp \left( - \frac{\delta_{s}}{\delta_{c}} \right) \right]
\]

(11)

where \( \epsilon = \exp(1) \); \( \sigma_{n} \), material tensile strength; and \( \delta_{c} \), critical displacement. As illustrated in Fig. 2, the relationship between the traction and displacement jump, upon loading, follows the form:

\[
t = \frac{\partial \phi}{\partial \delta_{n}} = \epsilon \sigma_{n} \frac{\delta_{s}}{\delta_{c}} \exp \left( - \frac{\delta_{s}}{\delta_{c}} \right)
\]

(12)

and for unloading and reloading, the traction can be obtained with the following expression:

\[
t = \left( \frac{\delta_{c}}{\delta_{n}} \right)
\]

(13)

where subscript \( A \) indicates a point where unloading starts to occur in the cohesive law (see Fig. 2). The unloading path follows toward the origin of the cohesive law. The cohesive fracture energy is defined by

\[
G_{c} = \int_{0}^{\infty} t d\delta = \epsilon \sigma_{n} \delta_{c}
\]

(14)
order to capture the time-temperature dependency, the complex modulus is measured over a range of frequencies (25, 10, 5, 1, 0.5, and 0.1 Hz) and temperatures (−10 to 60°C). In this simulation, the Young’s modulus is taken as 14.2 GPa based upon complex modulus testing of the mixture at 10°C and 1 Hz.

Two experimental fracture properties of material strength and cohesive fracture energy are evaluated as material inputs into the CZM. The tensile strength of the asphalt concrete is currently determined at −10°C. The first-failure tensile strength determined from the IDT test is defined as the material strength. The procedure for determining the first-failure tensile strength is outlined in the AASHTO TP-9 specification (AASHTO 1996). A material strength of 3.56 MPa was experimentally determined from the IDT test. Fig. 3(a) illustrates the IDT test setting.

The cohesive fracture energy of the asphalt concrete is the other fracture property required as an input into the CZM. The single-edge notched beam [SE(B)] test was used for determining the cohesive fracture energy. The crack-mouth opening displacement (CMOD) was increased at a linear rate to produce a stable postpeak fracture. The CMOD rate was determined by trial and error to produce a peak load at approximately 5 s into the test. Five seconds was selected as the time to peak load based on the AASHTO (1996) TP-9 procedure for determining the tensile strength. The cohesive fracture energy was determined by calculating the area under the load-CMOD curve and normalizing by the cross-sectional area of the beam. At −10°C, a value of 344 J/m² was obtained. A detailed procedure for specimen preparation and test controls of the SE(B) test is outlined by Wagoner et al. (2005). Fig. 3(b) shows the SE(B) test apparatus. Notice that both cohesive parameters, evaluated at −10°C, are employed in this study to investigate cracking mechanism occurring under freezing temperatures. For this temperature range, volumetric-cohesive viscoelastic effects are not directly considered.

Verification of CZM

In order to verify the numerical implementation of the aforementioned CZM (i.e., the potential-based exponential model) into the UEL of ABAQUS, a double cantilever beam (DCB) is adopted, because the DCB is well accepted by the fracture community and an analytical solution exists. Using linear elastic beam theory, the analytical solution for crack length in terms of the Young’s modulus (E), the end displacement (Δ), the beam height (H), and the cohesive fracture energy (Gc) is obtained (Anderson 1995) as

\[ a = \frac{4}{3} \sqrt{\frac{3EH^3\Delta^2}{4G_c}} \]  

Fig. 4 illustrates a schematic of the DCB geometry. To avoid shear effects in the beam, a relatively slender DCB of length \( L = 200 \text{ mm} \) and width \( W = 10 \text{ mm} \) is adopted. External displacement is applied to the node located at \( x = 0 \) and \( y = 0 \) upward and downward. Cohesive elements are inserted along the middle of the specimen. Two-dimensional plane strain elements and linear four-node cohesive elements are employed for the bulk material and cohesive material, respectively. To obtain material and cohesive parameters, a 9.5 mm nominal maximum-sized aggregate surface mixture is selected, which is used at the Greater Peoria Regional Airport. The Young’s modulus is taken as 14.2 GPa, based upon the aforementioned modulus tests. A Poisson’s ratio value of 0.35 is assumed, based upon previous experience with similar materials. The cohesive fracture energy and material strength for this mixture are 344 J/m² and 3.56 MPa, respectively. For the exponential model, 0.1\( \delta_c \) is defined as the crack tip location.

Fig. 5 illustrates a comparison between the numerical and analytical solutions. The abscissa indicates normalized crack length, \( a/L \), and the ordinate indicates the normalized crack opening displacement, \( \delta/\delta_c \). The numerical results show excellent agreement with the analytical solution. Notice that, even for both the initial stage and final stage of crack propagation, which are influenced by boundary conditions, both numerical and analytical results agree reasonably well.

Mode I Single-Edge Notched Beam [SE(B)] Test

In this section, utilizing the mode I SE(B) test, various important aspects of CZM are presented. First, sensitivity analysis to cohesive parameters of the material strength and cohesive fracture
energy is performed. Second, cohesive parameters of material strength and cohesive fracture energy are calibrated by comparing the numerical result with experimental results of the SE(B) test. Finally, justification of the cohesive element size is provided. Three different cohesive element sizes are chosen, and numerical results for each cohesive element size are compared to make sure that the element sizes chosen in the simulation are small enough to capture the nonlinear behavior occurring along the cohesive zone (Klein et al. 2001).

**Sensitivity Analysis with Respect to Fracture Energy and Material Strength**

Using the condition of small-scale yielding, Tvergaard and Hutchinson (1992) demonstrated that, in general, the influence of the shape of the traction separation law on the numerical responses is relatively weak as compared to other cohesive parameters, e.g., material strength ($\sigma_c$) and cohesive fracture energy ($G_c$). Thus, in this section, the sensitivity analysis to cohesive parameters of material strength and fracture energy is carried out to explore the influence of cohesive parameters, i.e., $\sigma_c = 3.56$ MPa and $G_c = 344 \text{ J/m}^2$ (Wagoner et al. 2005).

Fig. 6(a) illustrates a simply supported SE(B) with a length of 376 mm, height of 100 mm, and thickness of 75 mm. A mechanical notch is simulated, which extends 19 mm upward from the bottom edge of the beam. Displacement boundary conditions are imposed at the center of the top edge of the model. Figs. 6(b and c) show a finite-element configuration for the whole geometry and the center region of the specimen where the cohesive elements are inserted, respectively. Two-dimensional, four-noded cohesive elements are inserted along the center of the specimen. The bulk material is modeled as elastic, homogeneous, isotropic, and rate independent. These assumptions will be verified in a subsequent study. Given the low test temperature and short test duration, it is assumed that the bulk material can be adequately simulated with elastic materials. In this analysis, $E=14.2$ GPa and $\nu=0.35$. The Newton-Raphson method and plane strain conditions are adopted.

Fig. 7(a) illustrates the sensitivity of the $P$ versus CMOD curve to different fracture energies. Three different fracture energies, i.e., $1.2G_c$, $G_c$, and $0.8G_c$, are employed with a constant value of critical strength. As the fracture energy is increased, the area of the $P$ versus CMOD curve is increased and the maximum load is increased as well. This result is intuitive because, as the intrinsic fracture energy used in the CZM is increased, more global fracture energy is released, which is indicated by an increased area under the $P$ versus CMOD curve. The softening trend, however, seems insensitive to the magnitude of fracture energy. Fig. 7(b) shows the sensitivity of the $P$ versus CMOD curve to different critical strengths, $1.2\sigma_c$, $\sigma_c$, and $0.8\sigma_c$. As the critical strength is increased, the maximum load is increased, while the area of the curve remains almost constant.

**Calibration of Cohesive Parameters**

In the nonlinear cohesive constitutive model, cohesive fracture energy and material strength are two important parameters. These parameters are measured directly from the experiments and reflect the actual viscoelastic heterogeneous material. However, quasi-static homogeneous materials are assumed in the numerical modeling. Thus, the parameters of the CZM model are calibrated by fitting the present numerical results into experimental results in order to take into account these differences between the actual and numerical models.

A first-order calibration of material strength and cohesive fracture energy was accomplished by matching the present numerical results with experimental SE(B) test results (see Fig. 8). Relatively small calibration shifts of the cohesive parameters, i.e., $0.7G_c=0.7 \times 344 \text{ J/m}^2$ and $1.1\sigma_c=1.1 \times 3.56$ MPa, are required to bring the simulated results into reasonable comparison with the measured results. Notice that, for the rest of simulation, the calibrated cohesive parameters are employed.

**Selection of Intrinsic Cohesive Element Size**

When the concept of the cohesive zone model is combined with the discrete finite-element method, a numerical issue as to the sensitivity of the size of the cohesive element to the numerical solution arises. This is due to the fact that the cohesive zone is
reaction increases for different cohesive element sizes. A general rule in choosing the element size is that there should be at least three elements or so along the fracture process zone. For some specific brittle materials, the fracture process zone can be estimated theoretically (Rice 1968) as

\[ I_e = \frac{\pi}{8} \frac{E}{1 - \nu^2} \frac{G_c}{\sigma_{ave}^2} \]  

(16)

where \( G_c \) = cohesive fracture energy; and \( \sigma_{ave} \) = measure of material strength in an average sense. However, this estimation is not valid for materials such as asphalt concrete, which is quasi-brittle and viscoelastic, because Eq. (16) is evaluated based on the assumption that energy is absorbed in a very thin cohesive zone without any consideration of viscoelastic effects. Thus, from a numerical point of view, although viscoelastic effects are not directly considered, it is crucial to make sure that the cohesive element size chosen is not sensitive to artifacts of the numerical solution.

Three different cohesive element sizes, i.e., 0.1, 0.2, and 1.0 mm, are employed. To illustrate that the cohesive element size chosen in this study is objective (i.e., somehow independent of a particular numerical solution), a local quantity, e.g., \( \delta_{25} \), and global quantities, i.e., CMOD and total dissipated energy due to fracture, are evaluated and compared. The \( \delta_{25} \), which is measured from a gauge length of 25 mm spanning the original crack tip, is introduced for operational definition of the crack tip opening displacement (CTOD) with the following advantages: It is a local quantity near the crack tip, and it can be applied to any cracked specimen due to direct and easy measurement of CTOD. The proposed \( \delta_{25} \) measurement is inspired by the work by Schwalbe and Cornec (1991) and Schwalbe (1995), who proposed the insightful \( \delta_5 \) concept. Notice that the original concept of \( \delta_5 \) was developed and has been applied for fine-grain-sized materials like steel (Castrodaza et al. 2004). However, due to the coarse microstructure of asphalt concrete (e.g., aggregate sizes ranging from 4.75 to 19 mm in this study), a \( \delta \)-type evaluation on the order of 25 mm is more appropriate, leading, for instance, to the \( \delta_{25} \) definition. A numerical investigation of \( \delta_{25} \) will be provided in the next example [a schematic is provided in Fig. 9(a)]. Theoretical verification and experimental validation for the proposed \( \delta_{25} \) will be addressed in subsequent investigations.

Fig. 7. Sensitivity of \( P \) versus CMOD curve to: (a) different fracture energy; (b) different material strength (\( \sigma_c = 3.56 \text{ MPa} \) and \( G_c = 344 \text{ J/m}^2 \))

Fig. 8. Comparison between experimental result and numerical result with calibrated parameters
The geometry, boundary conditions, material properties, and nonlinear numerical scheme used here are the same as those used in the section that discussed the calibration of cohesive parameters. Thus, notice that calibrated cohesive parameters are employed. Fig. 9

Fig. 9. Comparison of: (a) P versus CMOD and $\delta_{25}$; (b) CMOD versus total dissipated fracture energy for different cohesive element sizes: $l_1=0.1$ mm, $l_2=0.2$ mm, and $l_3=1.0$ mm ($l_i$ introduces a length-scale in the problem)

The geometry, boundary conditions, material properties, and nonlinear numerical scheme used here are the same as those used in the section that discussed the calibration of cohesive parameters. Thus, notice that calibrated cohesive parameters are employed. Fig. 9(a) illustrates $P$ (applied force) versus displacement curves in which both CMOD and $\delta_{25}$ are plotted together. Fig. 9(b) shows the consumption of the cohesive fracture energy as the crack propagates. The abscissa indicates the CMOD and the ordinate indicates the total dissipated fracture energy. Due to accumulation of the cohesive fracture energy, this shows an increasing trend of the total dissipated fracture energy with an increase of the CMOD. Both the global and local responses as a function of different cohesive element sizes are nearly identical, demonstrating that the cohesive elements chosen in this particular investigation are small enough to be insensitive to numerical artifacts.

Mixed-Mode SE(B)

Using the calibrated cohesive parameters and the cohesive element size 1.0 mm, a simulation of mixed-mode crack propagation in the SE(B) test is carried out. The cohesive elements are inserted over an area to allow cracks to propagate in any direction. Fig. 10(a) shows the geometry, boundary condition, and region enclosed by the dashed lines. The length, height, and thickness are 376, 100, and 75 mm, respectively. The crack tip is located at 65 mm to the left of and 19 mm above the center of the bottom edge. The displacement boundary condition is applied at the center of the top edge of the model. The cohesive elements are inserted over the area enclosed by the dashed lines. Fig. 10(b) illustrates finite-element discretization for the whole geometry and Fig. 10(c) shows finite-element mesh details of the region where the cohesive elements are inserted. The SE(B) structure is constructed by 5,810 three-noded triangular elements for bulk materials and by 1,010 four-noded linear elements for cohesive materials. Notice that, in order to avoid the numerical problem of nonconvergence, the mesh of Fig. 10(c) is tailored to the crack trajectory predicted by I-Franc2D (Kim 2003) and the Riks method (Crisfield 1980) is employed for this simulation.

Fig. 11(a) shows the final deformed shapes, which can be obtained with convergent solutions using the Riks method. A magnification factor of 10 is used to make the crack trajectory visible. We observed that, when the Newton-Raphson method is adopted, the cracks begin to grow, but eventually the solution diverges when a crack tip reaches around 40% of the height of the SE(B) specimen. The main reason for the nonconvergence in the CZM is that, during the quasi-static calculation, we often reach a point where the incremental solution jumps back and forth between two near equilibrium states (Tijssens et al. 2000). However, in this study, this numerical problem is not observed when the Riks method is employed, indicating its superior performance as compared to the Newton-Raphson method. Fig. 11(b) illustrates a comparison of the crack trajectory between the experimental and numerical results obtained using the Riks method. Green and blue lines indicate the crack trajectory of the front and back side of the specimen.
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References


lations and the applications of meshfree methods.” SAND 2001-8009, Sandia National Laboratories, Albuquerque, N.M.


