Identification of fracture properties for a cohesive model using digital image correlation

M. Alfano, G. Lubineau, A. Moussawi
Composite and Heterogeneous Materials Analysis and Simulations, King Abdullah University of Science and Technology, Kingdom of Saudi Arabia

G. H. Paulino
Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, USA

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Introduction and motivations (1/4)

Mechanical testing

\[ \theta \approx 83^\circ \]

as produced

\[ 20^\circ \]

laser treated

\[ 1.5 \text{ mm} \]

\[ 25 \text{ mm} \]

\[ 100 \text{ mm} \]

\[ 45 \text{ mm} \]

\[ 40 \text{ mm} \]

\[ 100 \text{ mm} \]

\[ P \delta \]

\[ \text{laser treated} \]

\[ \text{grit-blasted} \]

Load (N)

Displacement (mm)

- Laser irradiated
- Fitting \((R^2=0.999)\)
- Grit blasted
- Fitting \((R^2=0.987)\)
- Error bands \(\pm 1\) s

Introduction and motivations (2/4)

PPR based cohesive model

\[ \Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[ \Gamma_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^\alpha \left( \frac{m}{\gamma} + \frac{\Delta_n}{\delta_n} \right)^m + (\phi_n - \phi_t) \right] \]
\[ \times \left[ \Gamma_t \left( 1 - \frac{\Delta_t}{\delta_t} \right)^\beta \left( \frac{n}{\beta} + \frac{|\Delta_t|}{\delta_t} \right)^n + (\phi_t - \phi_n) \right]. \]

\[ \begin{array}{c|c}
\text{Axis} & \text{Units} \\
\hline
\Psi & \text{N/m} \\
T_n, T_t & \text{MPa} \\
\Delta_n, \Delta_t & \text{\mu m} \\
\end{array} \]
\[ \Psi(\Delta_n) = \phi_n + \Gamma_n \left(1 - \frac{\Delta_n}{\delta_n}\right)^\alpha \left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^m \]

\[ T(\Delta_n) = \frac{\partial \Psi}{\partial \Delta_n} = \frac{\Gamma_n}{\delta_n} \left[m \left(1 - \frac{\Delta_n}{\delta_n}\right)^\alpha \left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^{m-1} - \alpha \left(1 - \frac{\Delta_n}{\delta_n}\right)^{\alpha-1} \left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^m \right] \]

\[ \Gamma_n = -\phi_n \left(\frac{\alpha}{m}\right)^m \]

\[ m = \frac{\alpha (\alpha - 1) \lambda_n^2}{1 - \alpha \lambda_n^2} \]

\[ \delta_n = \frac{\phi_n}{\sigma_{max}} \alpha \lambda_n (1 - \lambda_n)^{\alpha-1} \left(\frac{\alpha}{m} + 1\right) \cdot \left(\frac{\alpha \lambda_n + 1}{m}\right)^{m-1} \]

\[ X = \{\phi_n, \sigma_{max}, \lambda_n, \alpha\} \]

energy constant;
non-dimensional exponent;
final crack opening width;
unknown properties to be identified

Introduction and motivations (4/4)

Identification of bond toughness using the CZM

The response function is not affected by shape factor and slope indicator.

A global response is often obtained from experiments, however, it may have low sensitivity to certain cohesive properties.

The uniqueness of the obtained cohesive zone model is not guaranteed.

Although the cohesive models obtained using global data can yield satisfactory predictive capabilities in FEA simulations of fracture, the development of an alternative procedure is needed, e.g. to determine cohesive strength.

A solution may be provided by the original combination of experimental **full-field measurements** techniques and **inverse problems**.


**Forward versus inverse problem**

**Forward problem**

\[
(K_b + K_c(u, X))u = F_{ext}
\]

\[
K_b = \int_{\Omega} B^T E B \, d\Omega
\]

\[
K_{coh} = \int_{\Gamma_{coh}} N^T \frac{\partial T}{\partial \Delta u} N \, d\Gamma_{coh}
\]

\[
F_{ext} = \int_{\Gamma_{ext}} T_{ext} d\Gamma_{ext}
\]

**Principle of virtual work**

\[
\int_{\Omega} \sigma : \delta \epsilon \, d\Omega - \int_{\Gamma_{ext}} T_{ext} \cdot \delta \Delta u \, d\Gamma_{ext} + \int_{\Gamma_{coh}} T_{coh} \cdot \delta \Delta u \, d\Gamma_{coh} = 0
\]

\(\Omega\) : specimen domain  
\(\Gamma_{ext}\) : external boundary  
\(\Gamma_{coh}\) : cohesive surfaces  
\(\Delta u\) : cohesive surfaces opening displacement  
\(\sigma\) : stress tensor  
\(\epsilon\) : strain tensor
Forward versus inverse problem
Inverse problem (objective of the work)

\[ \hat{X} = \arg \min_{X \in \mathbb{R}^M} \{ \Pi = \sum_{i=1}^{m} \omega_i(X) \} \]

\[ \omega_i(X) = \frac{1}{(U_{max,i})^2} \sum_{j=1}^{n_n} [u_{exp} - u(X)]_j^2 \]

- \( m \): available measurement instants (load levels)
- \( n_n \): nodal displacements in the ROI

\[ u(X) = K^{-1} \hat{F}^{ext}(u_{exp} ; X) \]

\[ \hat{F}^{ext}(u_{exp} ; X) = F^{ext} - K_c(u_{exp} ; X)u_{exp} \]

Optimization algorithm?
Exploration algorithm based on the mechanism of natural selection and genetics: the strongest **individuals (chromosomes)** in a **population** survive and generate offsprings.

A **chromosome** represents a generic solution of the problem, in our context a set of cohesive fracture parameters ($X$):

\[
X \Phi_n \sigma_{\text{max}} \lambda_n \alpha
\]

**Basic steps of the GA**

1. Random generation of the **initial population** (individuals $X$) satisfying suitable restraint conditions (e.g. fracture energy must not be negative);
2. The chromosomes are evaluated, using some measures of fitness. We defined the following objective function (or cost function):

\[ \omega_i (X) = \frac{1}{(U_{\text{max},i})^2} \sum_{j=1}^{n_n} \left[ u_{\text{exp}} - u(X) \right]_j^2 \]

\[ \hat{X} = \arg \min_{X \in \mathbb{R}^M} \left\{ \Pi = \sum_{i=1}^{m} \omega_i (X) \right\} \]

- \( m \) : available measurement instants (load levels)
- \( n_n \) : nodal displacements in the ROI

3. Individuals for reproduction are firstly chosen based on their fitness

4. and some of them are processed by means of genetic operators (crossover and mutation) to create a new populations
5. New chromosomes, called \textit{offspring}, are formed by merging two chromosomes from current generation

\textbf{Crossover (type 1)}

\[
\begin{align*}
    X_1 & : \Phi_n, \sigma_{\text{max}}, \lambda_n, \alpha \\
    X_2 & : \Phi_n, \sigma_{\text{max}}, \lambda_n, \alpha \\
    X_3 & : \Phi_n, \sigma_{\text{max}}, \lambda_n, \alpha \\
    X_4 & : \Phi_n, \sigma_{\text{max}}, \lambda_n, \alpha \\
\end{align*}
\]

\textbf{Crossover (type 2)}

\[
\begin{align*}
    X_3 &= a \cdot X_1 + (1 - a) \cdot X_2 \\
    X_4 &= (1 - a) \cdot X_1 + a \cdot X_2 \\
    a & \in [0, 1]
\end{align*}
\]
5. New chromosomes are also formed by modifying a chromosome using a mutation operator

\[
\begin{align*}
X_1 & \quad \Phi_n \quad \sigma_{\text{max}} \quad \lambda_n \quad \alpha \\
\downarrow & \quad \sigma'_{\text{max}} = \sigma_{\text{max}} + r \cdot \Delta \sigma_{\text{max}} \\
X_2 & \quad \Phi_n \quad \sigma'_{\text{max}} \quad \lambda_n \quad \alpha
\end{align*}
\]

\[r \in [-1, 1], \Delta \sigma_{\text{max}} = \text{cost.}\]

6. The newly created population replace the old one and the process restarts.
Genetic algorithm
Fundamentals concepts

**Start**

- **Initial solutions**

**Population P(t)**
- Chromosomes:
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_1$
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_2$
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_3$

**Offsprings C(t)**
- Chromosomes:
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_1$
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_2$
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_{12}$
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_{21}$
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_j$
  - $(\phi_n, \sigma_{max}, \lambda_n, \alpha)_{j'}$

**Terminating condition?**
- **Yes**
  - **Best solution**
  - **Stop**
- **No**
  - **New population**
  - **Solution candidates (fitness computation)**
  - **Selection**
  - **Evaluation**
  - **Crossover**
  - **Mutation**

**Abaqus/Matlab interaction by means of Linux shell scripts**

- Read output file
- Solve non-linear fracture problem
- Write input file
Target applications and experimental set-up

current status

Material processing - DCB with metal and composites substrates

Ultrasonic Dispersion
MWCNT Dispersion
Hot press 15T/300°C
Specialized cutting devices
Composite laminates

Full-field experimental measurements

INSTRON Testing machine
High resolution CCD camera (PixelFly)
High intensity Led spotlight
QM100 long distance microscope (QUESTAR)
In-house developed DIC algorithm
