Pattern Gradation and Repetition with Application to High-Rise Building Design

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Skidmore, Owings & Merrill, LLP
Motivation: Functionally Graded Buildings

Unconstrained Problem

John Hancock Building

Patterns of Different Sizes

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Motivation: Functionally Graded Buildings

Pattern 1: Commercial

Pattern 2: Residential

Unconstrained Problem

Trump Tower

Groups of Patterns

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Outline

• Introduction and Motivation

• Topology Optimization Framework
  • Basic problem formulation

• Manufacturing constraints and pattern gradation
  • Uniform Density approach
  • CAMD approach
  • Projection scheme (length scale)

• Numerical Results
  • 2D
    • Graded thicknesses using Lagrange Multipliers
    • Building example in 3D

• Concluding Remarks
Topology Optimization Framework

• Minimum compliance problem in discrete form:

\[
\begin{align*}
\text{min:} & \quad c(\rho, u) \\
\text{s. t.:} & \quad K(\rho)u = f \\
& \quad \int_{\Omega} \rho \, dV \leq V_s
\end{align*}
\]

- Objective function
- Equilibrium constraint
- Volume constraint

• Solid Isotropic Material with Penalization (SIMP) model:

\[
E(x) = \rho(x)^p E^0, \quad p > 1
\]

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Manufacturing Constraints and Pattern Gradation

- Uniform Element Density Approach: design variables are coincident with element centroids (or nodes)

- CAMD Approach: design variables are nodal densities and shape functions used to obtain density throughout design domain

\[
\rho(x) = \sum_{e=1}^{n} \sum_{i=1}^{4} N_i^e(x) \rho_i^e
\]

- Update sensitivities:

\[
\frac{\partial c}{\partial \rho_d} = \sum \frac{\partial c}{\partial \rho_i^e} \frac{\partial \rho_i^e}{\partial \rho_d}
\]
Projection scheme with graded patterns

- Projection using element centroids or nodal densities as design variables

- Must use scaled projection for pattern gradation
Numerical Examples

- Graded cantilever building: Uniform Element Densities

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Pattern constraints | Design variables

Mesh: 80 x 240
cenal = 4, \( r_{\text{min}} = 1.2 \)
volume = 50%
Numerical Examples

- Patterns of Different Sizes: Uniform Element Densities

Mesh: 80 x 240
penal = 4, $r_{\text{min}} = 1.2$
Numerical Examples

- Multiuse Building
  w/ volume gradation

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Pattern 1: Commercial

Pattern 2: Residential

Unconstrained Volume (50%)

Mesh: 75 x 360
penal = 4, $r_{\text{min}} = 1.2$

With volume constraints
Virtual Work/Lagrange Multipliers for gradation in wall thicknesses

• Virtual Work

\[ W_i = \int_A \left[ N^T \delta \eta + M^T \delta \chi + V^T \delta \Gamma \right] dA \]

axial flexural shear

• Lagrange Method

\[ \Delta = \Sigma_{plates} \xi_j + \lambda (\Sigma t_j A_j - V) \]

• Optimal thickness

\[ t_i = \frac{1}{\Delta_{req}} \left( \frac{\nu_i}{A_i} \right)^{0.5} \Sigma_j \left( A_j \nu_j \right)^{0.5} \]

Virtual Work/Lagrange Multipliers for gradation in wall thicknesses

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Building Design Example

Skidmore, Owings & Merrill
Proposed Tower Design
Hong Kong


Proposed Topology Optimization Design using Pattern Repetition
1,728,000 design variables

N = 8
Mesh: 96 x 12 x 12
Volume = 50%
Concluding Remarks

• Manufacturing constraints in topology optimization allow for design of optimal buildings in terms of stiffness, cost, deflection, etc.

• Additional building design considerations, such as stability and nonlinear behavior are sources for future investigation.
Concluding Remarks

• The present approach may be extended for industry purposes by exploring computational expenses associated with non-coincident FEM displacement and design variable meshes to be used on a larger scale

• Future work includes optimization of large scale 3D problems using Topological Data Structure (TopS) integrated with finite element analysis and topology optimization

USNCCM X Presentation