Axial Deformation: Advanced Topics

1. Strain energy
2. Prestresses, prestrains and misfits
3. Axially Loaded Members in statically indeterminate problems
4. Thermal Effects

Rail Buckling due to Temperature - en.wikipedia.org
2. Misfits, Prestrains, Prestresses

- Suppose that a member of a structure is manufactured with its length slightly different from its prescribed length. Then the member will not fit into the structure in its intended manner, and the geometry of the structure will be different from what was planned. We refer to situations of this kind as misfits.

- Sometimes misfits are intentionally created in order to introduce strains into the structure at the time it is built. Because these strains exist before any loads are applied to the structure, they are called prestrains.

- Accompanying the prestrains are prestresses, and the structure is said to be prestressed.
2. Misfits, Prestrains, Prestresses

Concrete **compressive** strength: 20-40 MPa
Concrete **tensile** strength: 2-5 MPa

*If an internal compressive stress exists in concrete before a tensile load is applied, the concrete member will be able to resist more tension.*

**Principle of pre-stressed concrete:** apply compressive pre-stress in concrete by subjecting steel reinforcement bar to tensile stress.
4. **Thermal Effects**

**Free Thermal Expansion**
Solids expand due to a temperature increase.

$$\frac{\delta T}{L} = \alpha \times \Delta T = \alpha \times (T - T_{ref})$$

<table>
<thead>
<tr>
<th>Material</th>
<th>$\alpha$ (x10⁻⁶/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>23.1</td>
</tr>
<tr>
<td>Steel</td>
<td>11-13</td>
</tr>
<tr>
<td>Concrete</td>
<td>12</td>
</tr>
<tr>
<td>Glass</td>
<td>8.5</td>
</tr>
<tr>
<td>Oak</td>
<td>54</td>
</tr>
<tr>
<td>Liquid Water</td>
<td>69</td>
</tr>
</tbody>
</table>
4. Thermal Effects

Equivalent Mechanical Effect of Temperature:
The equivalent stress that would produce the same elongation as an increase of temperature $\Delta T$ is called the thermal stress:

$$\sigma_T = E\varepsilon_T = E\frac{\delta_T}{L} = E\alpha \times \Delta T$$

In steel, the equivalent thermal stress that would produce an elongation under an increase of temperature by $100^\circ C$ is

$$\sigma_T = 29,000 \text{ psi}$$
4. Thermal Effects

Total Elongation
If the body is loaded both mechanically and thermally, total elongation is the sum of mechanical and thermal elongation:

$$\delta = \delta_M + \delta_T = L \times \left( \frac{P}{EA} + \alpha \Delta T \right)$$
4. **Thermal Effects**

**Statically Indeterminate Problems**

In statically indeterminate problems, the body is not free to deform. For a temperature increase, reactions to thermal expansion at the supports will produce internal stress.

\[ 0 = \delta = (\delta_M)_{reaction} + \delta_T \]

\[ (\delta_M)_{reaction} = -L\alpha\Delta T \]

\[ \sigma_T = E \frac{\delta_M}{L} = -E\alpha\Delta T \]

- Equivalent compression for temperature increase
- Equivalent tension for temperature decrease
4. **Thermal Effects: Summary**

Statically Determinate Problems  
(free thermal deformation)

\[
\delta = \delta_M + \delta_T = L \times \left( \frac{P}{EA} + \alpha \Delta T \right)
\]

Statically Indeterminate Problems  
(hindered thermal deformation)

\[
\sigma = \sigma_M + \sigma_T = \frac{N}{A} - E\alpha \Delta T
\]