STRUCTURAL OPTIMIZATION: FROM CONTINUUM AND GROUND STRUCTURES TO ADDITIVE MANUFACTURING

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TABLE OF CONTENTS

1. INTRO & MOTIVATION

2. TRUSS LAYOUT OPTIMIZATION WITHIN A CONTINUUM

3. LATERAL BRACING SYSTEMS

4. 2D GROUND STRUCTURES
ZEGARD T, PAULINO GH (2014). "GRAND – GROUND STRUCTURE BASED TOPOLOGY OPTIMIZATION ON ARBITRARY 2D DOMAINS USING MATLAB." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, JUNE.

5. 3D GROUND STRUCTURES
ZEGARD T, PAULINO GH (XXXX). "GRAND3 – GROUND STRUCTURE BASED TOPOLOGY OPTIMIZATION ON ARBITRARY 3D DOMAINS USING MATLAB." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, IN PREPARATION.

6. ADDITIVE MANUFACTURING OF OPTIMAL STRUCTURES

7. SUMMARY & CONCLUSIONS
ROADMAP

INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
3D GROUND STRUCTURES
ADDITIVE MANUF. OF OPT. STRUCT.
SUMMARY & CONCLUSIONS
1) INTRO & MOTIVATION

- WHY USE STRUCTURAL OPTIMIZATION?

LIMITED RESOURCES  EXTREME STRUCTURES  FUNCTIONAL
1) INTRO & MOTIVATION

• WHY DISCRETE—CONTINUUM?
  – LIMITED MODELING CAPABILITY
  – REASONABLE SIMPLIFICATIONS OF REALITY

REAL FRAME

SIMPLIFIED FRAME MODEL
1) INTRO & MOTIVATION

- GRADIENT VS. NON-GRADIENT
1) INTRO & MOTIVATION

- STRUCTURAL OPTIMIZATION METHODS
1) INTRO & MOTIVATION

- POTENTIAL APPLICATIONS

ANCHOR POINT LOCATION  LATERAL BRACING SYSTEM  REINFORCEMENT LAYOUT
ROADMAP

INTRO & MOTIVATION

TRUSS LAYOUT IN A CONTINUUM
2) TRUSS LAYOUT WITHIN A CONTINUUM

• 3D BEAM WITH REINFORCEMENT

36X11X8 MESH (19008 TET10)
SLAB: \( L_X=10 \) \( L_Y=3 \) \( L_Z=2 \)
\( E=100 \) \( \nu =1/3 \)
CABLE: \( A_E=500 \)
LOAD: NORMAL ON TOP FACET
2) TRUSS LAYOUT WITHIN A CONTINUUM

- 3D BEAM WITH REINFORCEMENT
2) TRUSS LAYOUT WITHIN A CONTINUUM

- 3D BEAM WITH REINFORCEMENT
3) LATERAL BRACING SYSTEMS

- EXAMPLES OF BRACED BUILDINGS

- JOHN HANCOCK CENTER (CHICAGO, IL)
- ALCOA BUILDING (SAN FRANCISCO, CA)
- BUILDING IN PDTE. RIESCO AVENUE (SANTIAGO, CHILE)
3) LATERAL BRACING SYSTEMS

• OTHER APPLICATIONS

STAGE HIRE

CONSTRUCTION SCAFFOLDING
3) LATERAL BRACING SYSTEMS

• BRACING POINT UPPER BOUND

<table>
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<tr>
<td>3D</td>
<td>0.625H</td>
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</table>

TWO-DIMENSIONAL BRACE

THREE-DIMENSIONAL BRACE
ROADMAP

INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
4) GROUND STRUCTURES IN 2D

• TRUSS LAYOUT OPTIMIZATION IS HIGHLY NONLINEAR

4) GROUND STRUCTURES IN 2D

- TRUSS LAYOUT OPTIMIZATION IS HIGHLY NONLINEAR
4) GROUND STRUCTURES IN 2D

• MAIN IDEA:
CONVERT A GEOMETRY AND SIZE OPTIMIZATION TO A SIZING-ONLY PROBLEM

• PLASTIC FORMULATION:

\[
\begin{align*}
\min_{\mathbf{a}} & \quad V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} & \quad \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& \quad -\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{if} \quad a_i > 0 \\
& \quad a_i \geq 0 \quad i = 1, 2 \ldots N_b
\end{align*}
\]
4) GROUND STRUCTURES IN 2D

\[ \min_{a} \quad V = l^T a \]
\[ \text{s.t.} \quad B^T n = f \]
\[ -\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{if} \quad a_i > 0 \]
\[ a_i \geq 0 \quad i = 1, 2 \ldots N_b \]

\[ \text{VANISHING CONSTRAINT} \]

• MULTIPLYING THE INEQUALITY BY CROSS-SECTIONAL AREA

\[ \min_{a} \quad V = l^T a \]
\[ \text{s.t.} \quad B^T n = f \]
\[ -\sigma_C a_i \leq n_i \leq \sigma_T a_i \]
4) GROUND STRUCTURES IN 2D

\[
\begin{align*}
\min_{a} \quad & V = l^T a \\
\text{s.t.} \quad & B^T n = f \\
& -\sigma_C a_i \leq n_i \leq \sigma_T a_i
\end{align*}
\]

- INTRODUCING SLACK VARIABLES

\[
\begin{align*}
  n_i + 2 \frac{\sigma_0}{\sigma_C} s^-_i &= \sigma_T a_i \\
  -n_i + 2 \frac{\sigma_0}{\sigma_T} s^+_i &= \sigma_C a_i \\
  \sigma_0 &= (\sigma_T + \sigma_C) / 2
\end{align*}
\]

\[
\begin{align*}
  a_i &= \frac{s^+_i}{\sigma_T} + \frac{s^-_i}{\sigma_C} \\
  n_i &= s^+_i - s^-_i
\end{align*}
\]

\[
\begin{align*}
\min_{s^+, s^-} \quad & V = l^T \left( \frac{s^+}{\sigma_T} + \frac{s^-}{\sigma_C} \right) \\
\text{s.t.} \quad & B^T (s^+ - s^-) = f \\
& s^+_i, s^-_i \geq 0
\end{align*}
\]
4) GROUND STRUCTURES IN 2D

• REMARKS
  – DESIGN VARIABLES DOUBLED: $S^+$ AND $S^-$
  – NO MORE VANISHING CONSTRAINT
  – DIFFERENT LIMITS IN TENSION AND COMPRESSION
  – LINEAR PROGRAM


$$\begin{align*}
\min_{s^+, s^-} & \quad V = l^T \left( \frac{s^+}{\sigma_T} + \frac{s^-}{\sigma_C} \right) \\
\text{s.t.} & \quad B^T (s^+ - s^-) = f \\
& \quad s_i^+, s_i^- \geq 0
\end{align*}$$
4) GROUND STRUCTURES IN 2D

- SIZING OF A HIGHLY INTERCONNECTED AND REDUNDANT TRUSS

NODES 404
ELEMS 200
LEVEL 5
4) GROUND STRUCTURES IN 2D

- SIZING OF A HIGHLY INTERCONNECTED AND REDUNDANT TRUSS

BARS 23,201
4) GROUND STRUCTURES IN 2D

• UNIQUE SOLUTION — NO COLLINEAR BARS

GIVEN $\sigma_T = 1$ AND $P = 1$

\[ a_1 = 1.0 \quad a_2 = a_3 = 0.0 \]
\[ a_1 = 0.0 \quad a_2 = a_3 = 1.0 \]
\[ a_1 = 0.5 \quad a_2 = a_3 = 0.5 \]
4) GROUND STRUCTURES IN 2D

- HIGHLY INTERCONNECTED TRUSS
  - CONNECTIVITY GENERATION

TRUSS MEMBERS
AT THIS CONNECTION LEVEL

CONNECTION LEVEL: DRO
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - BASE MESH
GROUND STRUCTURE METHOD

• EXAMPLE
  – CONNECTIVITY: LEVEL 1
4) GROUND STRUCTURES IN 2D

• EXAMPLE
  – CONNECTIVITY: LEVEL 2
4) GROUND STRUCTURES IN 2D

• EXAMPLE
  – CONNECTIVITY: LEVEL 3
4) GROUND STRUCTURES IN 2D

• EXAMPLE
  – CONNECTIVITY: LEVEL 4
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 5
4) GROUND STRUCTURES IN 2D

• EXAMPLE
  – CONNECTIVITY: LEVEL 5
4) GROUND STRUCTURES IN 2D

- THERE CANNOT BE BARS EVERYWHERE
  - DEFINE ZONES WHERE NO BARS CAN BE
4) GROUND STRUCTURES IN 2D

- INTERSECTION TESTS FROM VIDEO-GAME AND COMPUTER GRAPHICS INDUSTRY
4) GROUND STRUCTURES IN 2D

• RESTRICTION ZONE PRIMITIVES
  – CIRCLE
  – SEGMENT (LINE)
  – RECTANGLE
  – POLYGON

• CAN BE COMBINED...
4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER

MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597
4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER

MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597
4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER
  
  Iteration 00
  
  28,256 BARS

MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597
4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER
4) GROUND STRUCTURES IN 2D

- HOOK PROBLEM

DOMAIN & BCs

GROUND STRUCTURES

72,589 BARS

DENSITY-BASED METHOD

10,000 ELEMS

4) GROUND STRUCTURES IN 2D

- FLOWER PROBLEM
4) GROUND STRUCTURES IN 2D

• FLOWER PROBLEM

69,400 BARS
ROADMAP

INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
3D GROUND STRUCTURES
5) GROUND STRUCTURES IN 3D

- **BASE-MESH DEFINITION**
  - GROUND STRUCTURE ALGORITHM SUPPORTS ANY CONVEX POLYTOPE
  - IMPLEMENTATION IS RESTRICTED TO 7 ELEMENTS: MESH GENERATION AND PLOTTING PURPOSES
5) GROUND STRUCTURES IN 3D

• RESTRICTION PRIMITIVES:

- TRIANGLE
- QUAD
- BOX
- SPHERE
- DISC
- CYLINDER
- ROD
- SURFACE
5) GROUND STRUCTURES IN 3D

• TORSION BALL PROBLEM

MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597
5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

\[ V_{opt} = 2M \log \left( \tan \left( \frac{\pi}{4} + \frac{\phi_F}{2} \right) \right) \left[ \frac{1}{\sigma_T} + \frac{1}{\sigma_C} \right] \]
5) GROUND STRUCTURES IN 3D

• TORSION BALL PROBLEM

MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597
5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM
5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM
5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

Iteration 000

268,636 BARS
5) GROUND STRUCTURES IN 3D

- TORSION BALL
  IMPROVING THE BASE MESH: SPHERICAL COORDINATES

![Diagram of a torsion ball with annotations for radii $r_o$, $r_m$, and $r_i$ and a mesh structure on the right side.]
5) GROUND STRUCTURES IN 3D

- TORSION BALL
IMPROVING THE BASE MESH: SPHERICAL COORDINATES
5) GROUND STRUCTURES IN 3D

• OTHER KNOWN SOLUTIONS?
5) GROUND STRUCTURES IN 3D

• MORE APPLIED PROBLEMS?

5) GROUND STRUCTURES IN 3D

• MORE APPLIED PROBLEMS?

5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?

4,100 BARS
5) GROUND STRUCTURES IN 3D

• MORE APPLIED PROBLEMS?
ROADMAP
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- INTRODUCTION TO DENSITY-BASED TOPOLOGY OPTIMIZATION

CANNONDALE CAPO
(URBAN COMMUTER BIKE)

BIKE DOMAIN AND LOADS

6) ADDITIVE MANUF. OF OPT. STRUCTS.

• INTRODUCTION TO DENSITY-BASED TOPOLOGY OPTIMIZATION

6) ADDITIVE MANUF. OF OPT. STRUCTS.

• DENSITY-BASED (NESTED) FORMULATION:
  – USING A DENSITY FILTER\(^1\)
  – MODIFIED SIMP\(^2\)

\[
\begin{align*}
\min_{\rho} & \quad J(\rho, \mathbf{u}(\rho)) \\
\text{s.t.} & \quad \bar{\rho} = H\rho \\
& \quad \sum_{i}^{N_e} \bar{\rho}_i v_i - (f)(V_0) \leq 0 \\
& \quad g_i(\rho, \mathbf{u}(\rho)) \leq 0 \quad i = 1 \ldots N_e \\
& \quad 0 \leq \rho_j \leq 1 \quad j = 1 \ldots N_e \\
& \quad E_k(\bar{\rho}_k) = E_{\text{min}} + \bar{\rho}_k^p (E_0 - E_{\text{min}}) \quad k = 1 \ldots N_e \\
\end{align*}
\]

with \(K(\bar{\rho}) \mathbf{u} = \mathbf{f}\)

\(^1\) BOURDIN B (2001) "FILTERS IN TOPOLOGY OPTIMIZATION." INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143–2158


\(^3\) ZHOU M, ROZVANY G (1991) "THE COC ALGORITHM, PART II: TOPOLOGICAL, GEOMETRICAL AND GENERALIZED SHAPE OPTIMIZATION." COMP METH APPL MECH ENGRG 89:309–336

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- **DENSITY-BASED (NESTED) FORMULATION:**
  - USING A DENSITY FILTER\(^1\)
  - MODIFIED SIMP\(^2\)

\[
\begin{align*}
\min_{\rho} & \quad J(\rho, u(\rho)) \\
\text{s.t.} & \quad \bar{\rho} = H\rho \\
& \quad \sum_{i}^{N_{c}} \bar{\rho}_{i} v_{i} - (f)(V_{0}) \leq 0 \\
& \quad g_{i}(\rho, u(\rho)) \leq 0 \quad i = 1 \ldots N_{c} \\
& \quad 0 \leq \rho_{j} \leq 1 \quad j = 1 \ldots N_{e} \\
& \quad E_{k}(\bar{\rho}_{k}) = E_{\min} + \bar{\rho}_{k}^{p}(E_{0} - E_{\min}) \quad k = 1 \ldots N_{e} \\
\end{align*}
\]

with \(K(\bar{\rho}) u = f\)

\(^{1}\) BOURDIN B (2001) “FILTERS IN TOPOLOGY OPTIMIZATION.” INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143–2158


6) ADDITIVE MANUF. OF OPT. STRUCTS.

- FILTERS IN DENSITY-BASED FORMULATION:
  - SENSITIVITY FILTER (1-FIELD)
  - DENSITY FILTER (2-FIELDS)
  - PROJECTION FILTER (3-FIELDS)

UNFILTERED (CHECKERBOARD)  FILTERED

USED IN THIS WORK

REVIEW ON FILTERING:
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- CONVOLUTION (BLURRING) OF THE DENSITY FIELD

\[ \bar{\rho} = H \rho \]

with \[ H_{ij} = \frac{h(i, j) v_j}{\sum_k^{N_e} h(i, k) v_k} \]

\[ h(i, j) = \begin{cases} [r_{min} - \text{dist}(i, j)]^q & \text{for } r_{min} - \text{dist}(i, j) > 0 \\ 0 & \text{otherwise} \end{cases} \]
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE–SUPPORTED CANTILEVER BEAM
  \( L_x = 3, \ L_y = L_z = 1, \ Q = 1, \ R = 5 \) AND \( \text{VOLFRAC} = 10\% \)

559,872 DVs FOR \( \frac{1}{2} \)
(1,119,744 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-SUPPORTED CANTILEVER BEAM
  \( L_x = 3, \ L_y = L_z = 1, \ Q = 1, \ R = 5 \) AND \( \text{VOLFRAC} = 10\% \)

Iteration 000  Penal = 3.00
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER BEAM
  \(L_x=3, \ L_y=L_z=1\)
  \(VOLFRAC=10\%, \ R=6, \ Q=1\) AND \(P=3\)

559,872 DVs FOR ½
(1,119,744 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER BEAM
  \( L_x = 3, \ L_y = L_z = 1, \ \text{Volfrac} = 10\%, \ R = 6, \ Q = 1 \) \text{AND} \ P = 3

Iteration 000  Penal = 3.00
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER
  \(L_x=3, L_y=L_z=1\)
  \(\text{VOLFRAC}=10\%, \ R=6, \ Q=1\ AND \ P=3\)

559,872 DVs FOR \(\frac{1}{2}\)
\((1,119,744\ TOTAL)\)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER
  \[ L_x=3, \ L_y=L_z=1 \]
  \[ \text{VOLFRAC}=10\%, \ R=6, \ Q=1 \ \text{AND} \ P=3 \]

559,872 DVs FOR \( \frac{1}{2} \)
(1,119,744 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- FILTER’S WEIGHTS FOR A REGULAR MESH $R_{\text{min}}=1.3$, $Q=1$ AND ELEM SIZE IS $L=1$

TWO-DIMENSIONS

THREE-DIMENSIONS ($H_{ii} = 0.4194$)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

IDEA: WHAT EXPONENT \( q \) MAKES \( H_{ii}^{(2D)} = H_{ii}^{(3D)} \)?

\[
q^{(3D)} = \log(r_{min}) + \frac{17}{20} q^{(2D)} + \frac{4}{57} q^{(2D)} r_{min} + \frac{4}{87} r_{min}
\]
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER
  $L_x=3$, $L_y=L_z=1$
  VOLFRAC=10%, $R=6$, $Q=3$ AND $P=3$

559,872 DVs FOR $\frac{1}{2}$
(1,119,744 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER
  \( L_X = 3, \ L_Y = L_Z = 1 \)
  \( VOLUME = 10\%, \ R = 6, \ Q = 3 \ AND \ P = 3 \)

559,872 DVs FOR \( \frac{1}{2} \)
(1,119,744 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER
  DENSITY FILTER: $R=6$

LINEAR DENSITY FILTER

CUBIC DENSITY FILTER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  \[ L_x = XXX, \quad L_y = L_z = YYY, \quad Q = ZZZ, \quad R = 5 \quad \text{AND} \quad \text{VOLFRAC} = 10\% \]
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  \[ L_x = 25, \ L_y = L_z = 5 \]
  \[ \text{VOLFRAC} = 10\%, \ R = 5, \ Q = 3 \ \text{AND} \ P = 3 \]

851,840 DVs FOR \( \frac{1}{4} \)
(3,407,360 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  \( L_x = 25, \quad L_y = L_z = 5 \)
  \( \text{VOLFRAC} = 10\%, \quad R = 5, \quad Q = 3 \quad \text{AND} \quad P = 3 \)

851,840 DVs FOR \( \frac{1}{4} \)
(3,407,360 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

• BRIDGE PROBLEM
  Lx=25, Ly=Lz=5
  VOLFRAC=10%, R=5, Q=3 AND P=3

851,840 DVs FOR ¼
(3,407,360 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SIMP’S POWER-LAW:

SIMP:
\[ E_i (\rho_i) = \rho_i^p E_0 \]
\[ 0 \leq \rho_{min} \leq \rho_j \leq 1 \]

MODIFIED SIMP:
\[ E_k (\rho_k) = E_{min} + \rho_k^p (E_0 - E_{min}) \]
\[ 0 \leq \rho_j \leq 1 \]

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- CONTINUATION OF “P” PARAMETER

ALLAIRE G, FRANCFORT G (1993) "A NUMERICAL ALGORITHM FOR TOPOLOGY AND SHAPE OPTIMIZATION." IN TOPOLOGY DESIGN OF STRUCTURES, SPRINGER
ALLAIRE G, KOHN R (1993) "TOPOLOGY OPTIMIZATION AND OPTIMAL SHAPE DESIGN USING HOMOGENIZATION." IN TOPOLOGY DESIGN OF STRUCTURES, SPRINGER
SIGMUND O, PETERSSON J (1998) "NUMERICAL INSTABILITIES IN TOPOLOGY OPTIMIZATION" STRUCTURAL OPTIMIZATION, 16(1):68–75
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  \( L_x = 25, \ L_y = L_z = 5, \ \text{VOLFRAC} = 10\%, \ R = 5, \ Q = 3 \) AND \( P = \text{[CONT]} \)

![Diagram of bridge problem]

Iteration 000  Penal = 2.00
6) ADDITIVE MANUF. OF OPT. STRUCTS.

• BRIDGE PROBLEM
  \( L_x=25, \ L_y=L_z=5, \ \text{VOLFRAC}=10\%, \ R=5, \ Q=3 \) AND \( P=[\text{CONT}] \)

851,840 DVs FOR ¼
(3,407,360 TOTAL)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- TOPSLICER: AN INSPECTOR/EXPORTER OF 3D DENSITY-BASED TOPOLOGIES
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- MANUFACTURING OF GROUND STRUCTURES

2D GROUND STRUCTURES

3D GROUND STRUCTURES
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- PROCEDURE
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- **X3D**: Royalty-free format for representing 3D computer graphics, managed by the Web3D Consortium.
6) ADDITIVE MANUF. OF OPT. STRUCTS.

• SHOW AND TELL:

• COLOR CODE

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<th>Description</th>
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</thead>
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</tr>
<tr>
<td>BLUE</td>
<td>2D GROUND STRUCTURES</td>
</tr>
<tr>
<td>RED</td>
<td>3D DENSITY METHOD</td>
</tr>
<tr>
<td>BLACK</td>
<td>APPLICATION-ORIENTED</td>
</tr>
</tbody>
</table>

• MANUFACTURED USING:
  - FDM: FUSED DEPOSITION MODELING
  - SLS: SELECTIVE LASER SINTERING
6) ADDITIVE MANUF. OF OPT. STRUCTS.

• SHOW AND TELL: TORSION BALL
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION BALL
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: CANTILEVER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

• SHOW AND TELL: CANTILEVER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- **SHOW AND TELL:**
  EDGE-LOADED 3D CANTILEVER (NO FIX)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL:
  EDGE-LOADED 3D CANTILEVER (NO FIX)
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: LOTTE TOWER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: LOTTE TOWER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: LOTTE TOWER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE ACHIEVING LARGE SCALES
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION–ORIENTED: CRANIOFACIAL RECONSTRUCTION

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: CRANIOFACIAL RECONSTRUCTION

Iteration 000    Penal = 1.50
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: CRANIOFACIAL RECONSTRUCTION
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: CRANIOFACIAL RECONSTRUCTION
7) SUMMARY AND CONCLUSIONS

- OPTIMIZATION:
  1. ESSENTIAL FOR SUSTAINABILITY
  2. CAN BE INCORPORATED INTO DESIGN TODAY
  3. GIVE A DESIGN, AND I WILL TRY TO MAKE IT BETTER
  4. DIFFERENT METHODS FOR DIFFERENT PROBLEMS
  5. YES, WE CAN MANUFACTURE THIS
  6. DESIGN GUIDED BY FUNCTIONALITY AND NOT JUST BEAUTY
7) SUMMARY AND CONCLUSIONS
7) SUMMARY AND CONCLUSIONS

- INTEGRATED DESIGN PROCESS: START TO FINISH
AKNOWLEDGEMENTS

• FULBRIGHT–CONICYT SCHOLARSHIP
• SKIMORE, OWINGS & MERRILL LLP
• CEE @ ILLINOIS
• ADVISER: GLAUCIO H. PAULINO
• RESEARCH GROUP
• PHD COMMITTEE
• MY FAMILY AND FRIENDS
2) TRUSS LAYOUT WITHIN A CONTINUUM

- SLAB WITH SUPPORTING CABLES

SELF WEIGHT OF SLAB
2) TRUSS LAYOUT WITHIN A CONTINUUM

- SLAB WITH SUPPORTING CABLES
2) TRUSS LAYOUT WITHIN A CONTINUUM

• TRUSS ELEMENT

\[ \mathbf{K}^* = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

\[ \mathbf{K}_e = \mathbf{T}_e^T \mathbf{K}^* \mathbf{T}_e \]

\[ \mathbf{T} = \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} \]

\[ d = \frac{1}{L} [x_2 - x_1, y_2 - y_1, z_2 - z_1] \]
2) TRUSS LAYOUT WITHIN A CONTINUUM

- SENSITIVITY W.R.T COORD ‘N’ OF NODE ‘J’

\[
\frac{\partial K_e}{\partial n_j} = \frac{\partial T_e^T}{\partial n_j} K^*_e T_e + T_e^T \frac{\partial K^*_e}{\partial L} \frac{\partial L}{\partial n_j} T_e + T_e^T K^*_e \frac{\partial T_e}{\partial n_j}
\]

\[
J_{(1)}(d) = \frac{1}{L} (d^T d - I) \quad J_{(2)}(d) = -J_{(1)}(d)
\]

\[
\frac{\partial L}{\partial n_1} = -d_n \quad \frac{\partial L}{\partial n_2} = d_n
\]

\[
\frac{\partial K^*}{\partial L} = -\frac{AE}{L^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
\]

2) TRUSS LAYOUT WITHIN A CONTINUUM

• DISPLACEMENTS “U” ANYWHERE IN \( \Omega \) USING FEM SHAPE FUNCTIONS

\[
u = Nu_c
\]

• CONFORMING COUPLING

– TRUSS MEMBER’S \( K_e \) IS COUPLED BY AN EQUIVALENT \( K_e^+ \) MATRIX

\[
u^T K_e u = u_c^T K_e^+ u_c
\]

\[
(Nu_c)^T K_e (Nu_c) = u_c^T K_e^+ u_c
\]

\[
u_c^T (N^T K_e N) u_c = u_c^T K_e^+ u_c
\]

\[
N^T K_e N = K_e^+
\]
2) TRUSS LAYOUT WITHIN A CONTINUUM

• PROOF-OF-CONCEPT: COMPLIANCE FORMULATION

\[
\begin{align*}
\min_{A,x} \quad & C = u^T Ku = u^T f \\
\text{s.t.} \quad & \sum_i A_i L_i \leq \bar{V} \\
\text{with} \quad & Ku = f
\end{align*}
\]

REQUIRED IF MEMBERS ARE ALSO SIZED

VOLUME CONSTRAINT IS NOT ALWAYS NECESSARY IN A GEOMETRY-ONLY OPTIMIZATION PROBLEM
2) TRUSS LAYOUT WITHIN A CONTINUUM

- 3X9 MESH (Q4 ELEMENTS)
  SLAB: $L_X=2$ $L_Y=0.8$ $E=100$ $\nu=0.3$
  CABLE: $AE=300$
  LOAD: $B=[0 \ -2]$

![Diagram of truss layout within a continuum](image)
2) TRUSS LAYOUT WITHIN A CONTINUUM

GLOBAL OPTIMUM

\( \partial C/\partial x = 0 \)

\( \partial C/\partial y = 0 \)
2) TRUSS LAYOUT WITHIN A CONTINUUM

SO WHAT NOW?

GLOBAL OPTIMUM

\frac{\partial C}{\partial x} = 0

\frac{\partial C}{\partial y} = 0
2) TRUSS LAYOUT WITHIN A CONTINUUM

- GAUSSIAN BLUR:
  - CONVOLUTION WITH A GAUSSIAN FUNCTION

IDEA:

- LETS “BLUR” THE FIRST DERIVATIVE!
2) TRUSS LAYOUT WITHIN A CONTINUUM

- ARBITRARILY SMOOTH CONVOLUTION?

\[ h(0) = 1 \]
\[ h(r \geq R) = 0 \]
\[ \left. \frac{dh}{dr} \right|_{r=R} = 0 \]

REQUIREMENTS

\[ h_1(r) = \begin{cases} 1 - \sin \left( \frac{r\pi}{2R} \right) & r \leq R \\ 0 & r > R \end{cases} \]
\[ h_2(r) = \begin{cases} \left( \frac{r}{R} \right)^2 - 2 \left( \frac{r}{R} \right) + 1 & r \leq R \\ 0 & r > R \end{cases} \]
2) TRUSS LAYOUT WITHIN A CONTINUUM

- CONVOLUTION-BASED SHAPE FUNCTIONS
  - PARTITION OF UNITY REQUIREMENT

\[ \tilde{N}_a = \frac{h(r_a)}{\sum_k h(r_k)} \]

\[ \begin{align*}
K_e^+ &= \tilde{N}^T K_e \tilde{N} \\
\frac{\partial K_e^+}{\partial n_j} &= \frac{\partial \tilde{N}^T}{\partial n_j} K_e \tilde{N} + \tilde{N}^T \frac{\partial K_e}{\partial n_j} \tilde{N} + \tilde{N}^T K_e \frac{\partial \tilde{N}}{\partial n_j}
\end{align*} \]
2) TRUSS LAYOUT WITHIN A CONTINUUM

• CONVOLUTION FUNCTION LOCALITY
  – SEARCH WITH BINARY, QUAD & OCT TREES
2) TRUSS LAYOUT WITHIN A CONTINUUM

FEM SHAPE FUNCTIONS

CONVOLUTION (R=0.5)

DETIAL Compliance [C]

DETIAL Compliance [C]
2) TRUSS LAYOUT WITHIN A CONTINUUM

CONVOLUTION (R=0.3)

CONVOLUTION (R=0.5)
2) TRUSS LAYOUT WITHIN A CONTINUUM

CONVOLUTION (R=0.3)

DETAIL Compliance [C]

PREVIOUS CASE

\( \frac{\partial C}{\partial x} = 0 \)
\( \frac{\partial C}{\partial y} = 0 \)
2) TRUSS LAYOUT WITHIN A CONTINUUM

- SLAB WITH SUPPORTING CABLES (8X21 Q9)

VIDEO

3X9 MESH (Q4 ELEMS)
SLAB: LX=2 LY=0.8 E=100 ν =0.3
CABLE: AE=300
LOAD: B=[0 -2]
2) TRUSS LAYOUT WITHIN A CONTINUUM

- DOUBLE CORBEL (UNSTRUCTURED MESH)

\[ q_n = 1.403 \text{ kips} \]

\[ q_n = 0.736 \text{ kips} \]

\[ q_t = 0.17 \text{ kips} \]
2) TRUSS LAYOUT WITHIN A CONTINUUM

- DOUBLE CORBEL (UNSTRUCTURED MESH)

VIDEO
2) TRUSS LAYOUT WITHIN A CONTINUUM

• DOUBLE CORBEL (UNSTRUCTURED MESH)
  – INTERPRETATION
3) LATERAL BRACING SYSTEMS

- UNIT BRACES IN 2 AND 3-DIMENSIONS

TWO-DIMENSIONAL BRACE

THREE-DIMENSIONAL BRACE
3) LATERAL BRACING SYSTEMS

• MULTIPLE STORIES – MULTIPLE BAYS
3) LATERAL BRACING SYSTEMS

- MULTIPLE STORIES - MULTIPLE BAYS

3D PANEL REPETITION
3) LATERAL BRACING SYSTEMS

• WHAT IS OPTIMAL?
  – LEAST WEIGHT
  – MINIMUM COMPLIANCE
  – SMALLEST DISPLACEMENT
  – OTHER...

• ASSUMPTIONS
  – ZERO CONNECTION COST
  – STATIC, LINEAR & ELASTIC
  – TRUSS MEMBERS
3) LATERAL BRACING SYSTEMS

- FORMULATIONS (1/2)
  - MINIMUM VOLUME
    \[
    \min_{A,x} \quad V = A^T L \\
    \text{s.t.} \quad \sigma_c \leq \sigma_i \leq \sigma_t \quad \forall i = 1 \ldots n_e \\
    \text{with} \quad Ku = f
    \]

- MINIMUM LOAD-PATH
  \[
  \min_{A,x} \quad Z = \sum_i |N_i| L_i \\
  \text{s.t.} \quad \sum_i A_i L_i \leq \bar{V} \\
  \text{with} \quad Ku = f
  \]
3) LATERAL BRACING SYSTEMS

• FORMULATIONS (2/2)
  – MINIMUM COMPLIANCE

\[
\begin{align*}
\min_{A,x} \quad & C = u^T Ku = u^T f \\
\text{s.t.} \quad & \sum_i A_i L_i \leq \bar{V} \\
\text{with} \quad & Ku = f
\end{align*}
\]

– MINIMUM DISPLACEMENT

\[
\begin{align*}
\min_{A,x} \quad & \Delta = u_j \\
\text{s.t.} \quad & \sum_i A_i L_i \leq \bar{V} \\
\text{with} \quad & Ku = f
\end{align*}
\]
3) LATERAL BRACING SYSTEMS

3D SYMMETRY:
BRACES ARE TWICE AS “EXPENSIVE” AS IN 2D
3) LATERAL BRACING SYSTEMS

- **MIN VOLUME ANALYTICAL SOLUTION**
  - 2D BRACE: $\alpha = 1$
  - 3D BRACE: $\alpha = 2$

\[
\mathcal{L} = \alpha A_1 L_1 + \alpha A_2 L_2 + A_3 H + \lambda_{11} (-A_1 \bar{\sigma} - N_1) + \lambda_{12} (-A_1 \bar{\sigma} + N_1) + ...
\]
\[
\lambda_{21} (-A_2 \bar{\sigma} - N_2) + \lambda_{22} (-A_2 \bar{\sigma} + N_2) + ...
\]
\[
\lambda_{31} (-A_3 \bar{\sigma} - N_3) + \lambda_{32} (-A_3 \bar{\sigma} + N_3)
\]

\[
\lambda_{11} = \alpha L_1 / \bar{\sigma} \quad \lambda_{12} = 0
\]
\[
\lambda_{21} = 0 \quad \lambda_{22} = \alpha L_2 / \bar{\sigma}
\]
\[
\lambda_{31} = 0 \quad \lambda_{32} = H / \bar{\sigma}
\]

\[
x = \frac{2\alpha + 1}{4\alpha} H
\]
\[
y = \frac{2\alpha - 1}{4\alpha} H
\]
3) LATERAL BRACING SYSTEMS

- **MIN COMPLIANCE ANALYTICAL SOLUTION**
  - 2D BRACE: $\alpha=1$
  - 3D BRACE: $\alpha=2$

\[ C = \frac{4P^2}{EB^2} \left[ \frac{L_1^3}{A_1} + \frac{L_2^3}{A_2} + \frac{L_3^3}{A_3} y^2 \right] \]

\[ \mathcal{L} = \frac{4P^2}{EB^2} \left[ \frac{L_1^3}{A_1} + \frac{L_2^3}{A_2} + \frac{L_3^3}{A_3} y^2 \right] + \lambda \left( \alpha A_1 L_1 + \alpha A_2 L_2 + A_3 H - \bar{V} \right) \]

\[ \lambda = \frac{4P^2 y^2}{EB^2 A_3^2} \]

\[ x = \frac{2\sqrt{\alpha} + 1}{4\sqrt{\alpha}} H \]

\[ y = \frac{2\sqrt{\alpha} - 1}{4\sqrt{\alpha}} H \]
3) LATERAL BRACING SYSTEMS

- ANALYTICAL SOLUTION FOR A SINGLE BAY
  - 2D BRACE: $\alpha=1$
  - 3D BRACE: $\alpha=2$

\[
x = \frac{2\alpha + 1}{4\alpha} H
\]

WEIGHT & LOAD-PATH

\[
x = \frac{2\sqrt{\alpha} + 1}{4\sqrt{\alpha}} H
\]

COMPLIANCE & DISPLACEMENT

<table>
<thead>
<tr>
<th>Height</th>
<th>Weight - Cost</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume</td>
<td>Load-Path</td>
</tr>
<tr>
<td>2D</td>
<td>0.75$H$</td>
<td>0.75$H$</td>
</tr>
<tr>
<td>3D</td>
<td>0.625$H$</td>
<td>0.625$H$</td>
</tr>
</tbody>
</table>
3) LATERAL BRACING SYSTEMS

• LIMIT CASE OF $\infty$ BAYS
3) LATERAL BRACING SYSTEMS

• LIMIT CASE OF $\infty$ BAYS
3) LATERAL BRACING SYSTEMS

- 3) LATERAL BRACING SYSTEMS

\[
\begin{align*}
N_{(c)i} &= \sigma_c A_i \\
N_{(t)i} &= \sigma_t A_i
\end{align*}
\]

\[
C = \sum_i \frac{|N_i|^2 L_i}{A_i E}
\]

\[
|N_i| / A_i = \bar{\sigma}
\]

\[
u = \begin{cases}
  u_1 \\
  \vdots \\
  u_{i-1} \\
  \Delta \\
  u_{i+1} \\
  \vdots
\end{cases}
\]

\[
f = \begin{cases}
  0 \\
  \vdots \\
  0 \\
  P \\
  i \\
  \vdots \\
  \vdots
\end{cases}
\]

---

<table>
<thead>
<tr>
<th>Height $x$</th>
<th>Weight - Cost</th>
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<tr>
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<tr>
<td>3D</td>
<td>0.625H</td>
<td>0.625H</td>
</tr>
</tbody>
</table>
3) LATERAL BRACING SYSTEMS

- OPTIMAL BRACING POINT FOR TWO-DIMENSIONAL BRACES

![Graphs showing the optimal bracing point for different stories and load ratios.](image-url)
3) LATERAL BRACING SYSTEMS

- OPTIMAL BRACING POINT FOR THREE-DIMENSIONAL BRACES

Graphs showing the relationship between \( x/H \) and number of bays for different values of \( P_z/P_x \) for 1, 2, and 3 stories.
3) LATERAL BRACING SYSTEMS

• GROUND STRUCTURE METHOD
  – WEIGHT MINIMIZATION WITH SYMMETRY

3) LATERAL BRACING SYSTEMS

• GROUND STRUCTURE METHOD
  – WEIGHT MINIMIZATION WITH SYMMETRY

2D RESULT

3D RESULT

3) LATERAL BRACING SYSTEMS

- GROUND STRUCTURE METHOD
  - WEIGHT MINIMIZATION WITH SYMMETRY

2D RESULT

3D RESULT

3) LATERAL BRACING SYSTEMS

- GROUND STRUCTURE METHOD
  - WEIGHT MINIMIZATION WITH SYMMETRY
3) LATERAL BRACING SYSTEMS

- GROUND STRUCTURE METHOD
  - WEIGHT MINIMIZATION WITH SYMMETRY

![Diagram](image)

2D RESULT

3D RESULT

4) GROUND STRUCTURES IN 2D

• BRACED TOWER
4) GROUND STRUCTURES IN 2D

- BRACED TOWER

11.5 MILLION BARS
5) GROUND STRUCTURES IN 3D

- DIRECT EXTENSION OF THE 2D METHOD
  - FORMULATION REMAINS UNMODIFIED

\[
\begin{align*}
\min_{s^+, s^-} & \quad V^* = \frac{V}{\sigma T} = \left\{ l^T \kappa l^T \right\} \left\{ \begin{array}{c} s^+ \\ \kappa l \end{array} \right\} \\
\text{s.t.} & \quad \left[ B^T \quad -B^T \right] \left\{ \begin{array}{c} s^+ \\ s^- \end{array} \right\} = f \\
& \quad s^+_i, s^-_i \geq 0 \\
& \quad \kappa = \frac{\sigma T}{\sigma C}
\end{align*}
\]
5) GROUND STRUCTURES IN 3D

• TORSION BALL PROBLEM
  – ERROR DECREASES FROM ~23% TO ~12%
6) ADDITIVE MANUF. OF OPT. STRUCTS.

• SHOW AND TELL: DIAMOND

NOTE: HALF-DOMAIN
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: DIAMOND
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: SHEAR BOX

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: SHEAR BOX
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION CONE
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION CONE
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION CYLINDER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION CYLINDER
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: PINWHEEL
6) ADDITIVE MANUF. OF OPT. STRUCTS.

• SHOW AND TELL: PINWHEEL