Potential-based Fracture Mechanics Using Cohesive Zone and Virtual Internal Bond Modeling

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Cohesive Zone Modeling

- Constitutive Relationship of Cohesive Fracture
  - Non-potential based-model vs. Potential based-model

- Computational Methods
  - Cohesive surface elements, enrichment functions, embedded discontinuities

Contents

- Introduction (Ch. 1)
- Potential-based Cohesive Model (Ch. 3)
- Quasi-Static Fracture (Ch. 5)
  - Particle/matrix debonding
- Dynamic Fracture Problems (Ch. 4, 6, 7)
  - Computational framework
  - Micro-branching and fragmentation
  - Mode I predefined crack, mixed-mode and branching
- Virtual Internal Pair-Bond (VIPB) Model (Ch. 2)
- Summary (Ch. 8)
Potentials for Cohesive Fracture

- **Needleman, A. (1987)**
  - Polynomial potential / linear shear interaction

- **Needleman, A. (1990)**
  - Exponential potential / periodic dependence

  - Generalized the potential (Exponential + Sinusoid)

- **Xu, X.P. and Needleman, A. (1993)**
  - Exponential potential (Exponential + Exponential)

- **Park, K., Paulino, G.H. and Roesler, J.R. (2009) – PPR**
  - Polynomial potential (Polynomial + Polynomial)

  - Needleman A. 1987, A continuum model for void nucleation by inclusion debonding, Journal of Applied Mechanics, 54, 525-531
  - Xu XP and Needleman, 1993, Void nucleation by inclusion debonding in a crystal matrix, Modeling Simulation Material Science Engineering, 1, 111-132.

Needleman A. 1987, A continuum model for void nucleation by inclusion debonding, Journal of Applied Mechanics, 54, 525-531

- **Polynomial Potential**

\[
\Psi(\Delta_n, \Delta_t) = \frac{27}{4} \sigma_{\text{max}} \delta_n \left\{ \frac{1}{2} \left( \frac{\Delta_n}{\delta_n} \right)^2 \left[ 1 - \frac{4}{3} \left( \frac{\Delta_n}{\delta_n} \right) + \frac{1}{2} \left( \frac{\Delta_n}{\delta_n} \right)^2 \right] + \frac{1}{2} \alpha_s \left( \frac{\Delta_t}{\delta_n} \right)^2 \left[ 1 - 2 \left( \frac{\Delta_n}{\delta_n} \right) + \left( \frac{\Delta_n}{\delta_n} \right)^2 \right] \right\}
\]

- **Cohesive Relationship**

\[
T_n = \frac{\partial \Psi}{\partial \Delta_n} = \frac{27}{4} \sigma_{\text{max}} \left\{ \left( \frac{\Delta_n}{\delta_n} \right) \left[ 1 - 2 \left( \frac{\Delta_n}{\delta_n} \right) + \left( \frac{\Delta_n}{\delta_n} \right)^2 \right] + \alpha_s \left( \frac{\Delta_t}{\delta_n} \right)^2 \left[ \left( \frac{\Delta_n}{\delta_n} \right) - 1 \right] \right\}
\]

\[
\Delta_n \leq \delta_n : \quad T_n = \frac{\partial \Psi}{\partial \Delta_n}
\]

\[
T_t = \frac{\partial \Psi}{\partial \Delta_t} = \frac{27}{4} \sigma_{\text{max}} \left\{ \alpha_s \left( \frac{\Delta_t}{\delta_n} \right) \left[ 1 - 2 \left( \frac{\Delta_n}{\delta_n} \right) + \left( \frac{\Delta_n}{\delta_n} \right)^2 \right] \right\}
\]

\[
\Delta_n > \delta_n : \quad T_n = T_t = 0
\]

\[
\phi_n = 9 \sigma_{\text{max}} \delta_n / 16
\]

\[
T_n(0, \delta_n / 3) = \sigma_{\text{max}}
\]

\(\alpha_s\): shear stiffness parameter

Shear dependence → linear
Displacement jump → small
\[ T_n = \frac{27}{4} \sigma_{\text{max}} \left\{ \frac{\Delta_n}{\delta_n} \left[ 1 - 2 \left( \frac{\Delta_n}{\delta_n} \right) + \left( \frac{\Delta_n}{\delta_n} \right)^2 \right] \right\} + \alpha_s \left( \frac{\Delta_t}{\delta_n} \right)^2 \left[ \left( \frac{\Delta_n}{\delta_n} \right) - 1 \right] \]

\[ T_t = \frac{27}{4} \sigma_{\text{max}} \left\{ \alpha_s \left( \frac{\Delta_t}{\delta_n} \right) \left[ 1 - 2 \left( \frac{\Delta_n}{\delta_n} \right) + \left( \frac{\Delta_n}{\delta_n} \right)^2 \right] \right\} \]

\[ \phi_n = 100 \text{ N/m} \]

\[ \sigma_{\text{max}} = 30 \text{ MPa} \]

\[ \alpha_s = 10 \]

<table>
<thead>
<tr>
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<tr>
<td>( \Psi )</td>
<td>( \Delta_n, \Delta_t )</td>
<td>N/m</td>
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<tr>
<td>( T_n, T_t )</td>
<td>MPa</td>
<td></td>
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<tr>
<td>( \Delta_n, \Delta_t )</td>
<td>( \mu \text{m} )</td>
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- Exponential Potential with Periodic Dependence

\[ \Psi(\Delta_n, \Delta_t) = \frac{\sigma_{\text{max}} e^{\delta_n}}{z} \left\{ 1 - \left[ 1 + \frac{z \Delta_n}{\delta_n} \right] - \beta_s z^2 \left[ 1 - \cos \left( \frac{2\pi \Delta_t}{\delta_t} \right) \right] \right\} \exp \left( -\frac{z \Delta_n}{\delta_n} \right) \]

- Motivation

  - Universal binding energy (Normal direction)

\[ E(a) = -(1 + \beta a) \exp(-\beta a) \]


  - Periodicity of the underlying lattice (Tangential direction)

- Cohesive Relationship

\[ T_n = \sigma_{\text{max}} e^{\left\{ \frac{z \Delta_n}{\delta_n} - \beta_s z^2 \left[ 1 - \cos \left( \frac{2\pi \Delta_t}{\delta_t} \right) \right] \right\} \exp \left( -\frac{z \Delta_n}{\delta_n} \right)} \]

\[ T_t = \sigma_{\text{max}} e^{\left\{ 2\pi \beta_s z \left( \frac{\delta_n}{\delta_t} \right) \sin \left( \frac{2\pi \Delta_t}{\delta_t} \right) \right\} \exp \left( -\frac{z \Delta_n}{\delta_n} \right)} \]
\[
T_n = \sigma_{\text{max}} e^{\frac{z \Delta_n}{\delta_n} - \beta_z z^2 \left[ 1 - \cos \left( \frac{2 \pi \Delta_t}{\delta_t} \right) \right]} \exp \left( -\frac{z \Delta_n}{\delta_n} \right)
\]

\[
T_t = \sigma_{\text{max}} e^{\frac{2 \pi \beta_z z \left( \frac{\delta_n}{\delta_t} \right) \sin \left( \frac{2 \pi \Delta_t}{\delta_t} \right)} \exp \left( -\frac{z \Delta_n}{\delta_n} \right)
\]

\[\phi_n = 100 \text{ N} / \text{m}\]

\[\sigma_{\text{max}} = 30 \text{ MPa}\]

\[z = 16 e / 9\]

\[\delta_n = \frac{z \phi_n}{e \sigma_{\text{max}}}\]

\[\delta_n = \delta_t\]

\[\beta_z = 1 / 2 \pi e z\]


**Cohesive Relationship**

- **Normal direction: Exponential** → $T_n = [B(\Delta_t)\Delta_n - C(\Delta_t)] \exp(-\Delta_n/\delta_n)$
- **Tangential direction: Sinusoid** → $T_t = A(\Delta_n) \sin \left(\frac{2\pi\Delta_t}{\delta_t}\right)$
- **Boundary Condition + Exact differential (Potential)**

\[
\begin{align*}
\int_0^\infty T_n(\Delta_n, 0) d\Delta_n &= 2\gamma_s = \phi_n \\
\int_0^{\delta_t/2} T_t(0, \Delta_t) d\Delta_t &= \gamma_{us} = \phi_t \\
C(0) &= 0
\end{align*}
\]

\[
\begin{align*}
\lim_{\Delta_n \to \infty} T_n(\Delta_n, \Delta_t) &= 0 \\
\lim_{\Delta_n \to \infty} T_t(\Delta_n, \Delta_t) &= 0 \\
\lim_{\Delta \to \frac{b}{2}} T_t(\Delta_n, \Delta_t) &= 0 \\
\lim_{\Delta \to \frac{b}{2}} T_n(\Delta_n, \Delta_t) &\neq 0
\end{align*}
\]

\[
T_n = \frac{\phi_n}{\delta_n^2} \Delta_n \exp\left(-\frac{\Delta_n}{\delta_n}\right)
\]

Universal bonding correlation.


Introduce additional condition $\Delta_n^*$ instead of $\lim_{\Delta \to \frac{b}{2}} T_n(\Delta_n, \Delta_t) = 0$

\[
T_n(\Delta_n^*, b/2) = \left[B\left(b/2\right)\Delta_n^* - C\left(b/2\right)\right] e^{-\Delta_n^*/\delta_n} = 0
\]

□ Solve PDE with BCs

\[
\frac{\partial T_n}{\partial \Delta t} = \frac{\partial T_t}{\partial \Delta_n}
\]

\[
T_n = [B(\Delta_t)\Delta_n - C(\Delta_t)] \exp(-\Delta_n/\delta_n)
\]

\[
T_t = A(\Delta_n) \sin \left( \frac{2\pi \Delta_t}{\delta_t} \right)
\]

\[
A(\Delta_n) = \frac{\pi \gamma_{us}}{\delta_t} - \frac{2\pi \gamma_s}{\delta_t} \left\{ q \left[ 1 - \exp \left( -\frac{\Delta_n}{\delta_n} \right) \right] - \left( \frac{q - r}{1 - r} \right) \frac{\Delta_n}{\delta_n} \exp \left( -\frac{\Delta_n}{\delta_n} \right) \right\}
\]

\[
B(\Delta_t) = \frac{2\gamma_s}{\delta_n^2} \left\{ 1 - \left( \frac{q - r}{1 - r} \right) \sin^2 \left( \frac{2\pi \Delta_t}{\delta_t} \right) \right\}
\]

\[
C(\Delta_t) = \frac{2\gamma_s}{\delta_n} \frac{r(1-q)}{1-r} \sin^2 \left( \frac{2\pi \Delta_t}{\delta_t} \right)
\]

\[
\Psi = 2\gamma_s + 2\gamma_s \exp \left( -\frac{\Delta_n}{\delta_n} \right) \left\{ \left[ q + \left( \frac{q - r}{1 - r} \right) \frac{\Delta_n}{\delta_n} \right] \sin^2 \left( \frac{2\pi \Delta_t}{\delta_t} \right) - \left[ 1 + \frac{\Delta_n}{\delta_n} \right] \right\}
\]

\[
E(a) = -(1+\beta a) \exp(-\beta a)
\]

\[
\int_0^\infty T_n(\Delta_n,0) \, d\Delta_n = 2\gamma_s = \phi_n
\]

\[
\int_0^{\delta_t/2} T_t(0,\Delta_t) \, d\Delta_t = \gamma_{us} = \phi_t
\]

\[
\lim_{\Delta n \to \infty} T_n(\Delta_n,\Delta_t) = 0 \quad \lim_{\Delta n \to \infty} T_t(\Delta_n,\Delta_t) = 0
\]

\[
C(0) = 0
\]

\[ \Delta_n^* \text{ is the value of } \Delta_n \text{ after shearing to the state } \Delta_t = b/2 \text{ under conditions of zero tension, } T_n = 0 \text{ (i.e. relaxed shearing)} \]

\[ r = \frac{\Delta_n^*}{\delta_n} = 0 \]

\[ T_n(\Delta_n^*, b/2) = \left[ B(b/2) \Delta_n^* - C(b/2) \right] e^{-\Delta_n^*/\delta_n} = 0 \]

Xu XP and Needleman, 1993, Void nucleation by inclusion debonding in a crystal matrix, Modeling Simulation Material Science Engineering, 1, 111-132.

- **Cohesive Relationship**

  - Normal direction: Exponential \( T_n = [B(\Delta_t) \Delta_n - C(\Delta_t)] \exp(-\Delta_n / \delta_n) \)
  
  - Tangential direction: Exponential \( T_t = A(\Delta_n) \frac{\Delta_t}{\delta_t} \exp(-\Delta_t^2 / \delta_t^2) \)
  
  - Boundary Condition

    \[
    \phi_n = \int_0^\infty T_n(\Delta_n,0) d\Delta_n \quad \phi_t = \int_0^\infty T_t(0,\Delta_t) d\Delta_t \quad C(0) = 0
    \]

    \[
    \lim_{\Delta n \to \infty} T_n(\Delta_n,\Delta_t) = 0 \quad \lim_{\Delta n \to \infty} T_t(\Delta_n,\Delta_t) = 0 \quad \lim_{\Delta t \to \infty} T_t(\Delta_n,\Delta_t) = 0 \quad \lim_{\Delta t \to \infty} T_n(\Delta_n,\Delta_t) \neq 0
    \]

    Introduce additional condition \( \Delta_n^* \) instead of \( \lim_{\Delta t \to \infty} T_n(\Delta_n,\Delta_t) = 0 \)

    \[
    \lim_{\Delta t \to \infty} T_n(\Delta_n^*,\Delta_t) = 0
    \]

    \[
    \Psi(\Delta_n,\Delta_t) = \phi_n + \phi_n \exp\left(-\frac{\Delta_n}{\delta_n}\right) \left\{ 1 - r + \frac{\Delta_n}{\delta_n} \right\} \left[ 1 - q + \frac{(r - q) \Delta_n}{(r - 1) \delta_n} \right] \exp\left(-\frac{\Delta_t^2}{\delta_t^2}\right)
    \]
\[ \lim_{\Delta t \to \infty} T_n(\Delta_n, \Delta_t) < 0 \]

\[ \phi_n = 100 N/m \]
\[ \phi_t = 200 N/m \]
\[ \sigma_{\max} = 30 \text{ MPa} \]
\[ \tau_{\max} = 40 \text{ MPa} \]

\[ r = \frac{\Delta_n}{\delta_n} = 0 \]
Remarks

- **Previous Potential**
  - Boundary condition is not symmetric $\Rightarrow \lim_{\Delta \to \infty} T_n(\Delta_n, \Delta_t) \neq 0$
  - Vague fracture parameter, $r$ (or $\Delta_n^*$) $\Rightarrow$ Okay if fracture energies are the same
  - Complete separation at infinity
  - Could not control initial slope $\Rightarrow$ Large compliance

- **Proposed Potential**
  - Expressed by a single function
  - Different fracture energy: $\phi_n, \phi_t$
  - Different cohesive strength: $\sigma_{\text{max}}, \tau_{\text{max}}$
  - Different cohesive law: $\alpha, \beta$
  - Different initial slope: $\lambda_n, \lambda_t$
Boundary Conditions

Various material behavior, i.e.

Plateau-type \(1 < \alpha, \beta << 2\)
Brittle material \(\alpha, \beta \approx 2\)
Quasi-brittle material \(\alpha, \beta >> 2\)

\[
\phi_n = \int_0^{\delta_n} T_n(\Delta_n, 0) d\Delta_n
\]
\[
\frac{\partial T_n}{\partial \Delta_n} \bigg|_{\Delta_n = \delta_{nc}} = 0
\]
\[
T_n(\delta_{nc}, 0) = \sigma_{\text{max}}
\]
\[
T_n(\delta_n, \Delta_t) = 0
\]
\[
T_n(\Delta_n, \delta_t) = 0
\]

\[
\phi_t = \int_0^{\delta_t} T_t(0, \Delta_t) d\Delta_t
\]
\[
\frac{\partial T_t}{\partial \Delta_t} \bigg|_{\Delta_t = \delta_{tc}} = 0
\]
\[
T_t(0, \delta_{tc}) = \tau_{\text{max}}
\]
\[
T_t(\Delta_n, \delta_t) = 0
\]
\[
T_t(\delta_n, \Delta_t) = 0
\]
PPR: Unified Mixed Mode Potential

\[ \Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[ \Gamma_n \left(1 - \frac{\Delta_n}{\delta_n}\right)^\alpha \left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^m + \langle \phi_n - \phi_t \rangle \right] \]

\[ \Gamma_t \left(1 - \frac{|\Delta_t|}{\delta_t}\right)^\beta \left(\frac{n}{\beta} + \frac{|\Delta_t|}{\delta_t}\right)^n + \langle \phi_t - \phi_n \rangle \]

- Energy Constants: \( \Gamma_n \) and \( \Gamma_t \)
- Exponents: \( m \) and \( n \)
- Characteristic length scales: \( \delta_n \) and \( \delta_t \)
- Shape parameters: \( \alpha \) and \( \beta \)

Fracture energy
Cohesive strength
Cohesive interaction shape
Initial slope

Softening Region

\[ T_n(\Delta_n, \Delta_t) = 0 \]

\[ \delta_n = \frac{\phi_n}{\sigma_{\text{max}}} \alpha \lambda_n (1 - \lambda_n)^{\alpha-1} \left( \frac{\alpha}{m} + 1 \right) \left( \frac{\alpha}{m} \lambda_n + 1 \right)^{m-1} \]

\[ T_n(\Delta_n, \bar{\delta}_t) = 0 \]

\[ f_t(\Delta_t) = \Gamma_t \left( 1 - \frac{\Delta_t}{\delta_t} \right)^{\beta} \left( \frac{n}{\beta} + \frac{\Delta_t}{\delta_t} \right)^{n} + \langle \phi_t - \phi_n \rangle = 0 \]

\[ T_t(\bar{\delta}_n, \Delta_t) = 0 \]

\[ f_n(\Delta_n) = \Gamma_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^{\alpha} \left( \frac{m}{\alpha} + \frac{\Delta_n}{\delta_n} \right)^{m} + \langle \phi_n - \phi_t \rangle = 0 \]

\[ T_t(\Delta_n, \delta_t) = 0 \]

\[ \delta_t = \frac{\phi_t}{\tau_{\text{max}}} \beta \lambda_t (1 - \lambda_t)^{\beta-1} \left( \frac{\beta}{n} + 1 \right) \left( \frac{\beta}{n} \lambda_t + 1 \right)^{n-1} \]
- Fracture energy
- Cohesive strength
- Cohesive interaction shape
- Initial slope

\[ \phi_n = 100 \text{ N/m} \]
\[ \sigma_{\text{max}} = 40 \text{ MPa} \]
\[ \lambda_n = 0.1 \]
\[ \beta = 1.3 \]
\[ \phi_t = 200 \text{ N/m} \]
\[ \tau_{\text{max}} = 30 \text{ MPa} \]
\[ \lambda_t = 0.2 \]
Extension for the **EXTRINSIC** Model

- **Correct Limit Procedure**
  - Limit of initial slope indicators in the potential
    \[
    \Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[ \Gamma_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^\alpha + (\phi_n - \phi_t) \right] \left[ \Gamma_t \left( 1 - \frac{|\Delta_t|}{\delta_t} \right)^\beta + (\phi_t - \phi_n) \right]
    \]
  - Energy constants: \( \Gamma_n \) and \( \Gamma_t \)
  - Characteristic length scales: \( \delta_n \) and \( \delta_t \)
  - Shape parameters: \( \alpha \) and \( \beta \)

- **Exclude elastic behavior** → **Extrinsic model**
- **Consider different fracture energy**: \( \phi_n, \phi_t \)
- **Describe different cohesive strength**: \( \sigma_{\text{max}}, \tau_{\text{max}} \)
- **Represent various cohesive shape**: \( \alpha, \beta \)
Softening Region

\[ T_n(\Delta_n, \Delta_t) = 0 \]
\[ \delta_n = \alpha \phi_n / \sigma_{\text{max}} \]

\[ T_n(\Delta_n, \bar{\delta}_t) = 0 \]
\[ \bar{\delta}_t = \delta_t - \delta_t \left( \frac{\phi_t - \phi_n}{\phi_t} \right)^{1/\beta} \]

\[ T_t(\Delta_n, \Delta_t) = 0 \]
\[ \bar{\delta}_n = \delta_n - \delta_n \left( \frac{\phi_n - \phi_t}{\phi_n} \right)^{1/\alpha} \]

\[ T_t(\Delta_n, \delta_t) = 0 \]
\[ \delta_t = \beta \phi_t / \tau_{\text{max}} \]
\[ \Psi(\Delta_n, \Delta_t) \]

\[ T_n(\Delta_n, \Delta_t) \]

\[ T_t(\Delta_n, \Delta_t) \]

Mode I

Mode II

\[ \phi_n = 100 \text{ N} / \text{m} \]
\[ \sigma_{\text{max}} = 40 \text{ MPa} \]
\[ \alpha = 5 \]
\[ \phi_t = 200 \text{ N} / \text{m} \]
\[ \tau_{\text{max}} = 30 \text{ MPa} \]
\[ \beta = 1.3 \]
Remarks

- Consistent boundary condition
- Fracture parameters
  - Energy, strength, shape, slope
- Intrinsic/Extrinsic cohesive zone modeling
- Path dependence of work-of-separation
  - Proportional path
  - Non-proportional path
- Unloading/reloading relationship is independent of the potential-based model
  - Coupled unloading/reloading relationship
  - Uncoupled unloading/reloading relationship
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Particle/Matrix Debonding

- Macroscopic Constitutive Relationship

- Micromechanics
  - Extended Mori-Tanaka Method
    - Hydro-static state
    - Interface debonding of cohesive constitutive relationship
  

- Computational Method
  - Cohesive surface elements
Finite Element Analysis

- Equi-biaxial Tension

- Assumptions
  - Reduced to 2D Problem
    - Plane strain
  - Quarter domain
    - Symmetry boundary conditions of unit cell
  - Poisson’s ratio = 0.25
  - Cohesive surface elements are inserted along the interface
Finite Element Mesh

- $E = 122 \text{ GPa}$
- $v = 0.25$
- $a = 2 \text{ mm}$
- $b = 2.29 \text{ mm}$
- $f = 60\%$
Effect of Cohesive Strength

- **Particle size: 100 μm**
- **Particle size: 2 mm**

**Micromechanics (Analytical)**

Debonding Process 1

• Fracture Parameters

<table>
<thead>
<tr>
<th>G (N/m)</th>
<th>$T_{max}$ (MPa)</th>
<th>Shape</th>
<th>Slope</th>
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<tbody>
<tr>
<td>5</td>
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<td>3</td>
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Debonding Process 2

- Fracture Parameters

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<td>30</td>
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<td>3</td>
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Debonding Process 3

- Fracture Parameters

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Debonding Process 4

- Fracture Parameters

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Debonding Process 5

• Fracture Parameters

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Debonding Process 6

- Fracture Parameters

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Debonding Process 7

Average strain (%)
Average stress (MPa)

- **Fracture Parameters**

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Topological Operators

- **Nodal Perturbation**

  ![Nodal Perturbation Diagram]

- **Edge-Swap**

  ![Edge-Swap Diagram]

Adaptive Mesh Refinement & Coarsening

- **Edge-Split**
  - Adaptive mesh refinement based on *a priori* knowledge

- **Vertex-Removal (or Edge-Collapse)**
  - Adaptive mesh coarsening based on *a posteriori* error estimation, i.e. root mean square of strain error

Topology-based Data Structure (TopS)

- Complete Topological Data & Reduced Representation
- Support for Adaptive Analysis
- Client-Server Architecture

- Separate computational mechanics from data representation

API functions

Client
(Analysis code)

Callback functions

Server
(TopS)

- W. Celes, G.H. Paulino, R. Espinha, 2005, A compact adjacency-based topological data structure for finite element mesh representation, IJNME 64(11), 1529-1556
Explicit Time Integration

Initialization: displacement, velocity, acceleration

for $n = 0$ to $n_{\text{max}}$ do

- Update displacement: $\mathbf{u}_{n+1} = \mathbf{u}_n + \Delta t \mathbf{u}_n + \Delta t^2 / 2 \ddot{\mathbf{u}}_n$
- Adaptive mesh coarsening (vertex-removal)
- Check the insertion of cohesive element (edge-swap)
- Update acceleration: $\ddot{\mathbf{u}}_{n+1} = \mathbf{M}^{-1}(\mathbf{R}_{n+1}^{\text{ext}} + \mathbf{R}_{n+1}^{\text{coh}} - \mathbf{R}_{n+1}^{\text{int}})$
- Update velocity: $\dot{\mathbf{u}}_{n+1} = \dot{\mathbf{u}}_n + \Delta t / 2(\ddot{\mathbf{u}}_n + \ddot{\mathbf{u}}_{n+1})$
- Update boundary conditions
- Adaptive mesh refinement (edge-split, nodal perturbation)

end
Micro-Branching Experiment

Computational Results

$\varepsilon_0 = 0.010$

$\varepsilon_0 = 0.012$

$\varepsilon_0 = 0.015$
Computational Results

- **Crack Velocity**

- **Energy Evolution (ε₀=0.015)**
Fragmentation Problem

\[ P(t) = p_0 e^{(-t/t_0)} \]
\[ (p_0 = 400 \text{ MPa, } t_0 = 100 \text{ µs}) \]
Computational Results
Mode I Pre-defined Crack Propagation

$\epsilon_0 = 0.036$

0.2 mm

unit thickness

Predefined path

- $E = 3.24 \text{ GPa}$
- $\nu = 0.35$
- $\rho = 1190 \text{ kg/m}^3$
- $G_I = 352 \text{ N/m}$
- $T_{\text{max}} = 324 \text{ MPa}$
Computational Results

- **Uniform Mesh Refinement**
  - 400x40 mesh grid
  - Element size: 5μm
  - 64000 elements, 128881 nodes

- **Adaptive Mesh Refinement**
  - 100x10 mesh grid
  - Element size: 20~5μm
  - 4448 elements, 9147 nodes
Computational Results (AMR)

![Energy evolution graph](attachment:image.png)

- Strain energy ($E_{\text{int}}$)
- Kinetic energy ($E_{\text{kin}}$)
- Fracture energy ($E_{\text{fra}}$)
- Total energy ($E_{\text{tot}}$)
- AMR: Strain energy
- AMR: Kinetic energy
- AMR: Fracture energy
- AMR: Total energy
Computational Results (AMR+C)
Mixed-Mode Crack Propagation

- Kalthoff-Winkler’s Experiments

Finite Element Mesh

- Initial Discretization

Animations (FE Mesh & Strain energy)
Computational Results

Previous results (X-FEM)


Crack Velocity
Branching Problem

$\sigma_0 = 1.5$ MPa

Elastic modulus: 32 GPa
Poisson’s ratio: 0.2
Density: 2450 kg/m$^3$
Fracture energy: 3 N/m
Cohesive strength: 12 MPa
Computational Results
Contents

- Introduction
- Potential-based Cohesive Model
- Quasi-Static Fracture
  - Particle/matrix debonding
- Dynamic Fracture Problems
  - Computational framework
  - Micro-branching and fragmentation
  - Mode I predefined crack, mixed-mode and branching
- Virtual Internal Pair-Bond (VIPB) Model
- Summary
Summary

- The potential-based constitutive model
  - Consistent boundary conditions
  - Physical fracture parameters

- Adaptive operators
  - Insertion of cohesive elements (Extrinsic model)
  - Nodal perturbation, Edge-swap
  - Edge-split, Vertex-removal

- Effective and efficient computational framework to simulate physical phenomena associated with quasi-static fracture, dynamic fracture, branching, and fragmentation problems
Contributions


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