Prediction of Concrete Fracture Mechanics Behavior and Size Effect using Cohesive Zone Modeling

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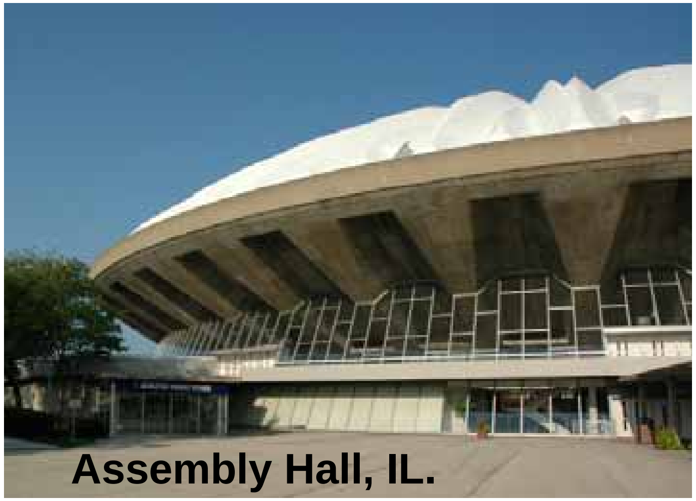
Concrete Structures

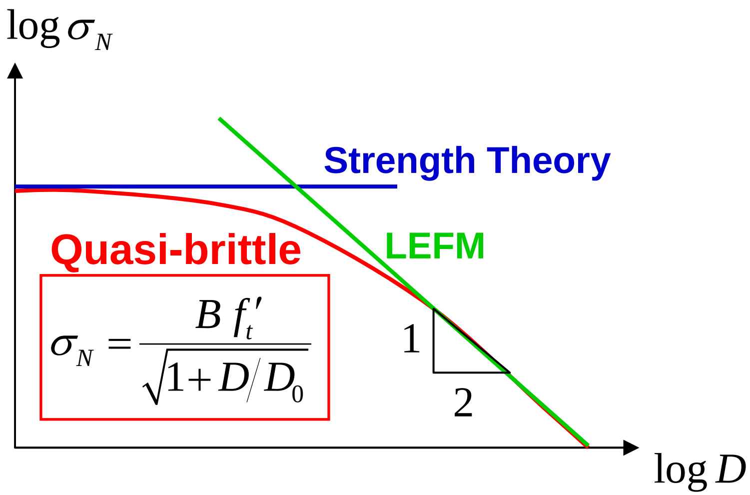


- Scaling
- Size Effect
- Fracture Mechanics

Fracture Mechanics Size Effect

Size Effect Method (SEM)

- Energy concept
- Equivalent elastic crack model
- Two size-independent fracture parameters: G_f and c_f



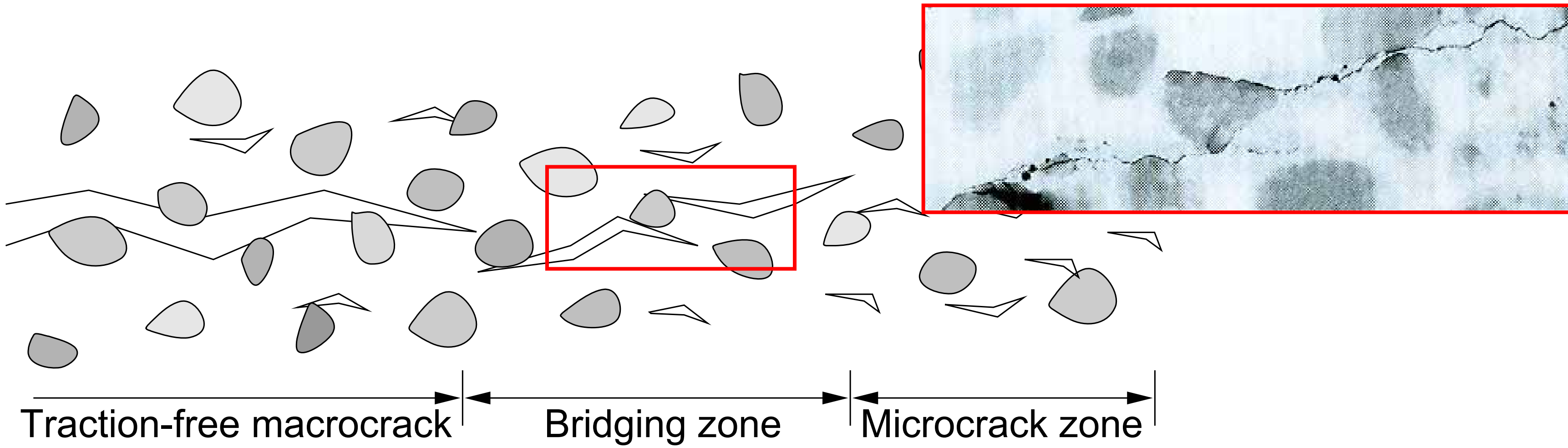
Bazant ZP, Kazemi MT. 1990, Determination of fracture energy, process zone length and brittleness number from size effect, with application to rock and concrete, *International Journal of Fracture*, 44, 111-131.

Two-Parameter Fracture Model (TPFM)

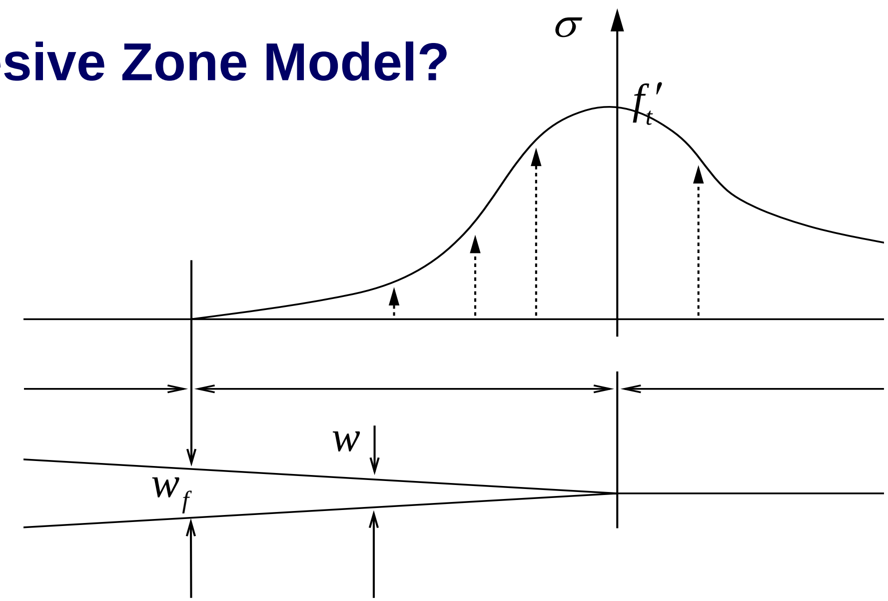
- Equivalent elastic crack model
- Two size-independent fracture parameters : K_I and $CTOD_c$

Jenq, Y. and Shah, S.P. 1985, Two parameter fracture model for concrete, *Journal of Engineering Mechanics*, 111, 1227-1241.

Mechanisms of Concrete Fracture



- **Why Cohesive Zone Model?**

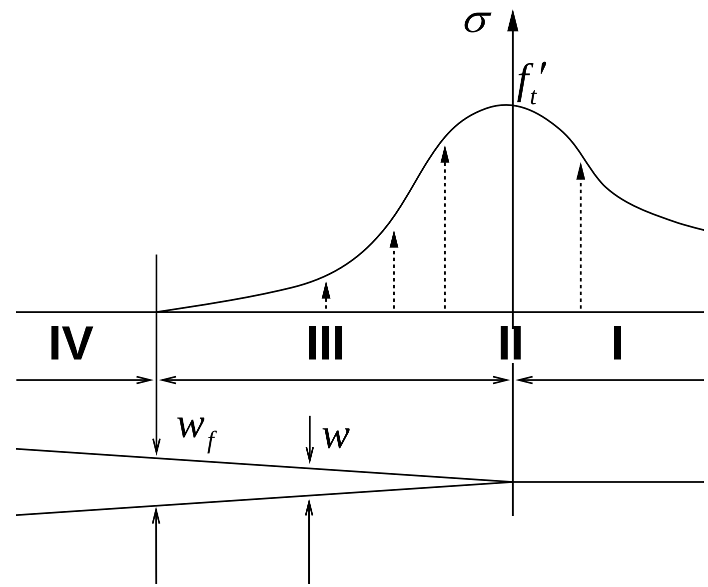


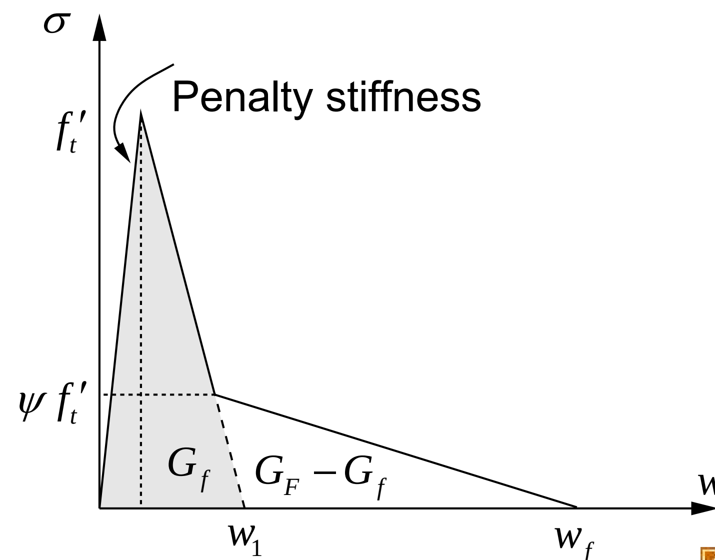
Outline

- Motivation
- Cohesive Zone Model for Concrete
- Finite Element Analysis Implementation
- Numerical Prediction of Three-point Bending Tests
- Size Effect
- Summary

Concept of Cohesive Zone Model

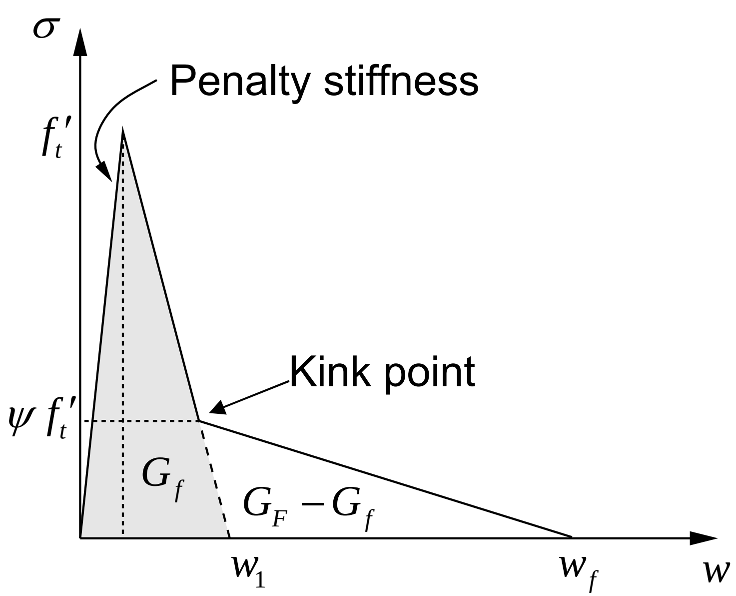
- **Stage I**
  - Elastic behavior
- **Stage II**
  - Crack initiation
  - Tensile strength
- **Stage III**
  - Non-linear cohesive law
  - Bi-linear softening curve for concrete
- **Stage IV**
  - Traction-free macro-crack





Determination of the Cohesive Law

Bi-linear softening curve



$$w_1 = \frac{2G_f}{f_t'}$$

$$w_f = \frac{2}{\psi f_t'}\left[G_F - (1-\psi)G_f\right]$$

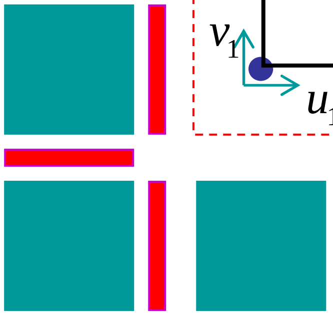
- **Cohesive strength : f_t'**
  - Splitting test
- **Initial fracture energy : G_f**
  - Size effect method (SEM)
  - Two-parameter fracture model (TPFM)
- **Total fracture energy: G_F**
  - Hillerborg's work-of-fracture method
- **The stress ratio of the kink point : ψ**
  - Peterson : 1/3
  - Wittmann : 0.25
  - Bazant : 0.15~0.33

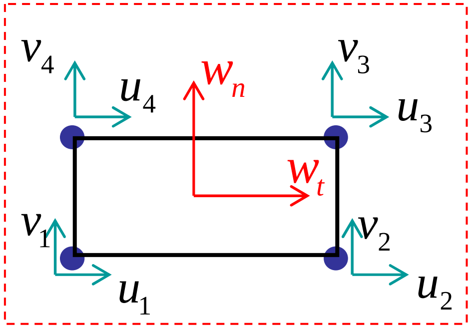
FEA Implementation

Principle of Virtual Work

- Virtual Internal Work = External Virtual Work

$$\int_{\Omega} \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} \, d\Omega + \int_{\Gamma_c} \delta \mathbf{w}^T \mathbf{T} \, d\Gamma_c = \int_{\Gamma} \delta \mathbf{u}^T \mathbf{F} \, d\Gamma$$





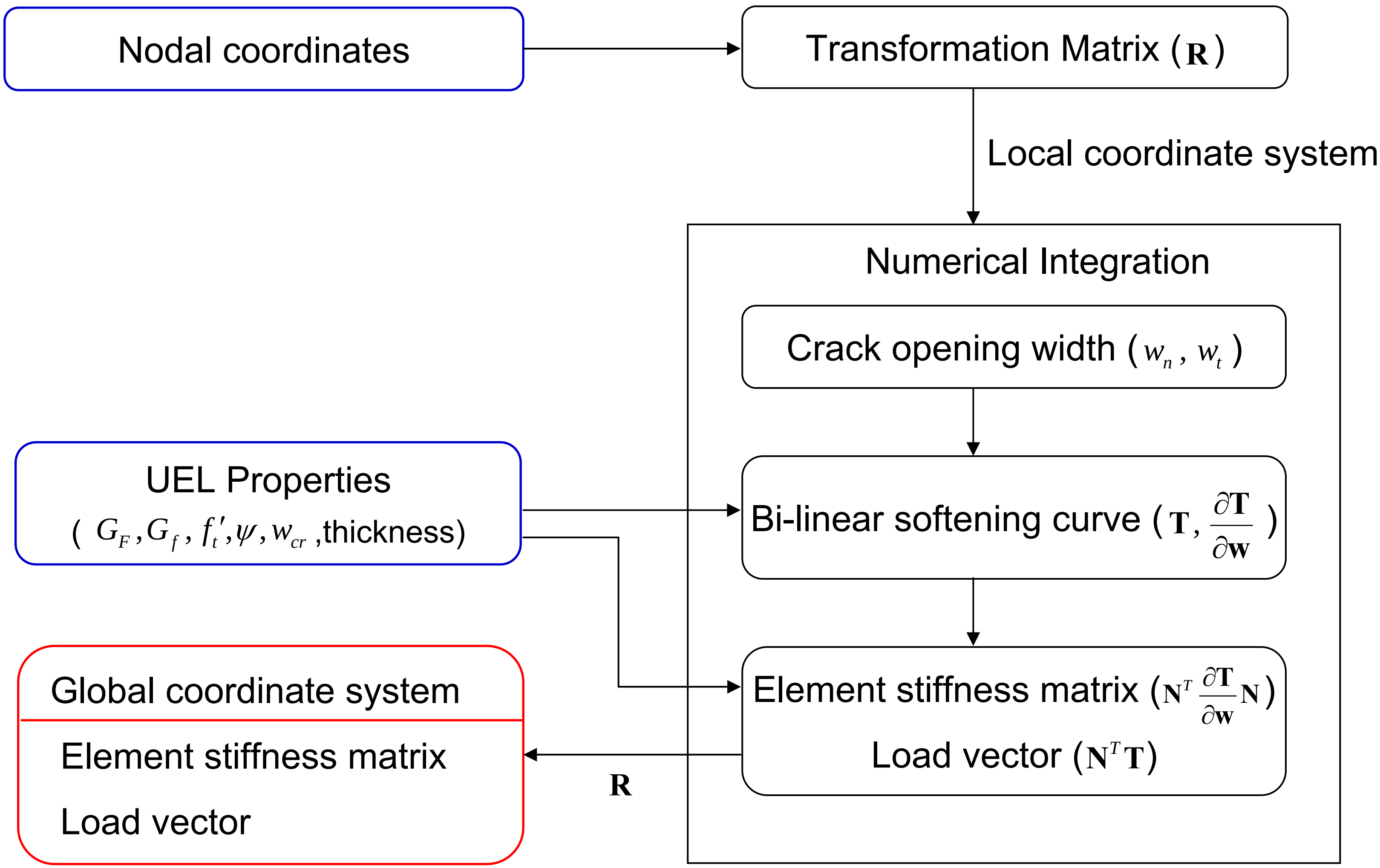
FEA Formulation

$$\left[\int_{\Omega} \mathbf{B}^T \mathbf{E} \, \mathbf{B} \, d\Omega + \int_{\Gamma_c} \mathbf{N}^T \frac{\partial \mathbf{T}}{\partial \mathbf{w}} \mathbf{N} \, d\Gamma_c \right] \mathbf{u} = \int_{\Gamma} \mathbf{P} \, d\Gamma$$

$$\frac{\partial \mathbf{T}}{\partial \mathbf{w}} = \begin{bmatrix} \dfrac{\partial T_t}{\partial w_t} & \dfrac{\partial T_t}{\partial w_n} \\ \dfrac{\partial T_n}{\partial w_t} & \dfrac{\partial T_n}{\partial w_n} \end{bmatrix}$$

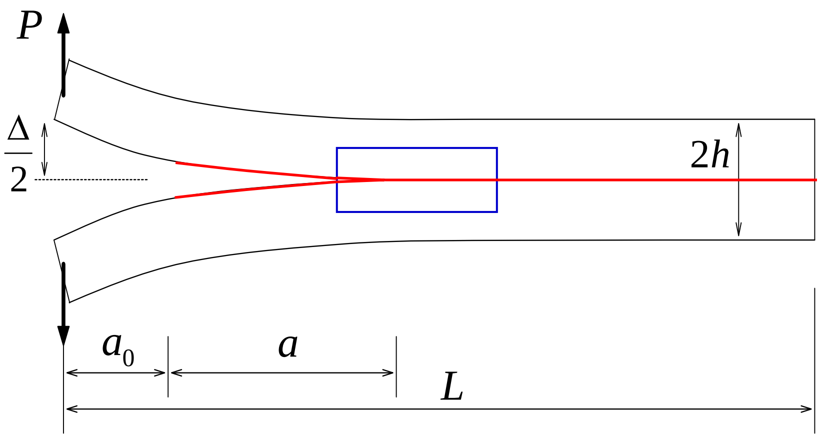
$$\frac{\partial T_n}{\partial w_n} = \begin{cases} f_t' / w_{cr} & (0 \le w_n < w_{cr}) \\ -f_t' / (w_1 - w_{cr}) & (w_{cr} \le w_n < w_k) \\ -\psi f_t' / (w_f - w_k) & (w_k \le w_n < w_f) \\ 0 & (w_n \ge w_f) \end{cases}$$

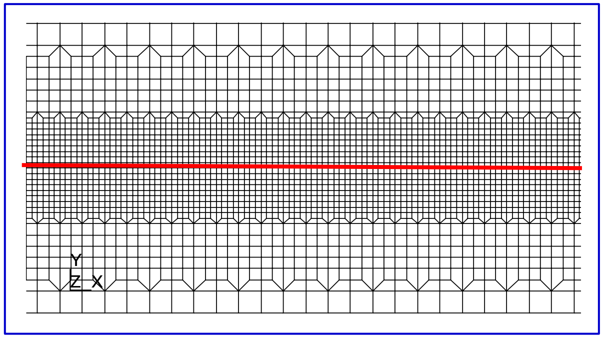
ABAQUS User Element (UEL)

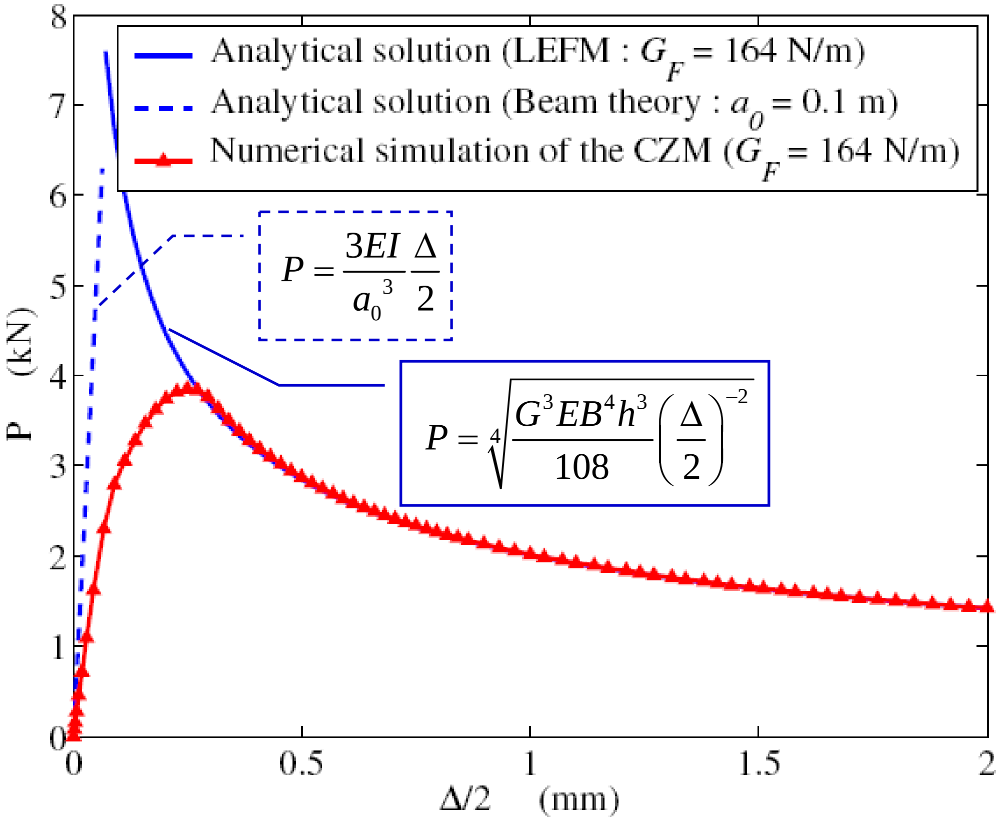


Numerical Verification

- **Double Cantilever Beam (DCB) Test**

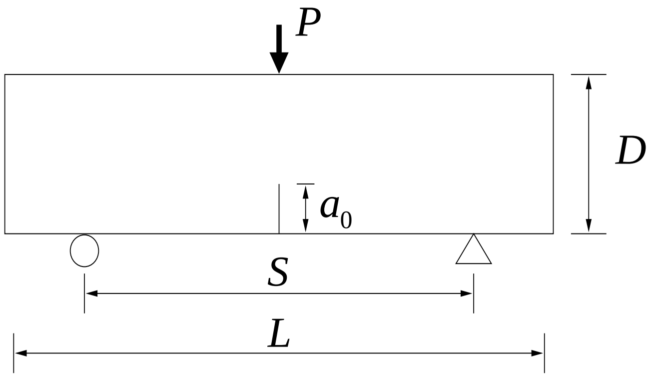






Three-Point Bending Test

- Obtain fracture parameters
- Compare load-CMOD curves
- Size effect



[mm]

Depth (D)	Span (S)	Length (L)	Notch (a_0)	Thickness (t)
63	250	350	21	80
150	600	700	50	80
250	1000	1100	83	80

Experimental Results

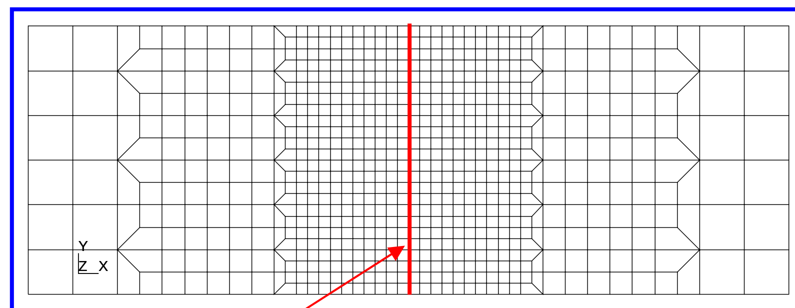
Fresh and Hardened Properties of the Concrete

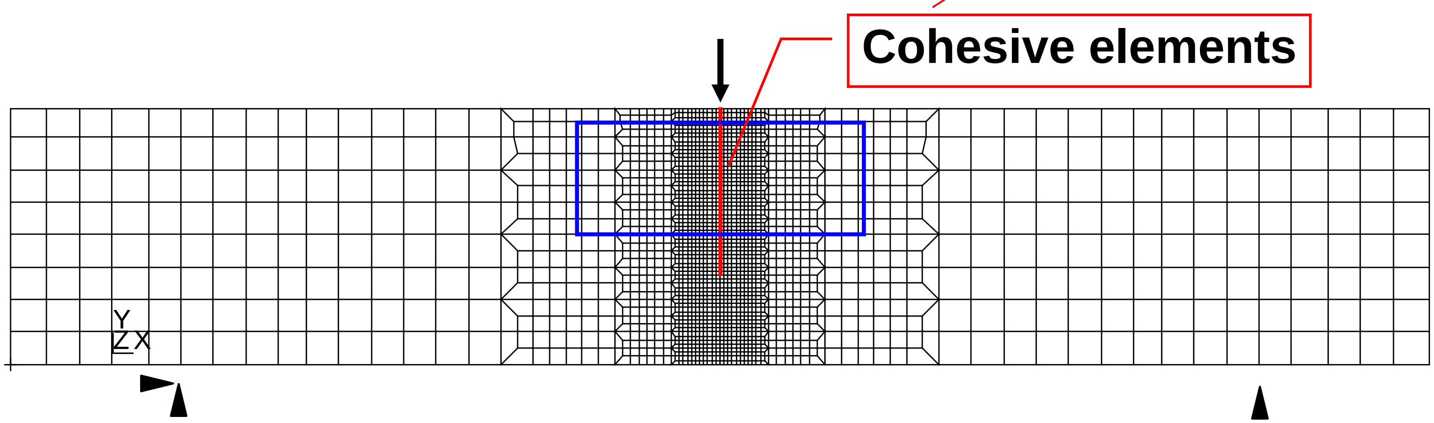
Fresh Concrete		Hardened Concrete	
Density	2403 kg/m^3	Compressive strength	58.3 MPa
Slump	100 mm	Split strength	4.15 MPa
Air content	2.8 %	Modulus of elasticity	32.0 GPa

Fracture Parameters

	Hillerborg	TPFM		SEM	
	G_F (N/m)	K_I (MPa m$^{1/2}$)	$CTOD_c$ (mm)	G_f (N/m)	c_f (mm)
B250-80a	193	1.261	0.0167	52.1	24.36
B250-80b	139	1.203	0.0181		
B250-80c	169	1.497	0.0319		
B150-80a	N/A	N/A	N/A		
B150-80b	170	1.086	0.0255		
B150-80c	159	0.983	0.0115		
B63-80a	N/A	N/A	N/A		
B63-80b	106	1.012	0.0159		
B63-80c	N/A	0.834	0.0115		
CB63-80a	123	1.130	0.0142		
CB63-80b	124	1.002	0.0075		
CB63-80c	123	1.293	0.0184		

Specimen Geometry and FE Mesh





Numerical Validation – Small Beam

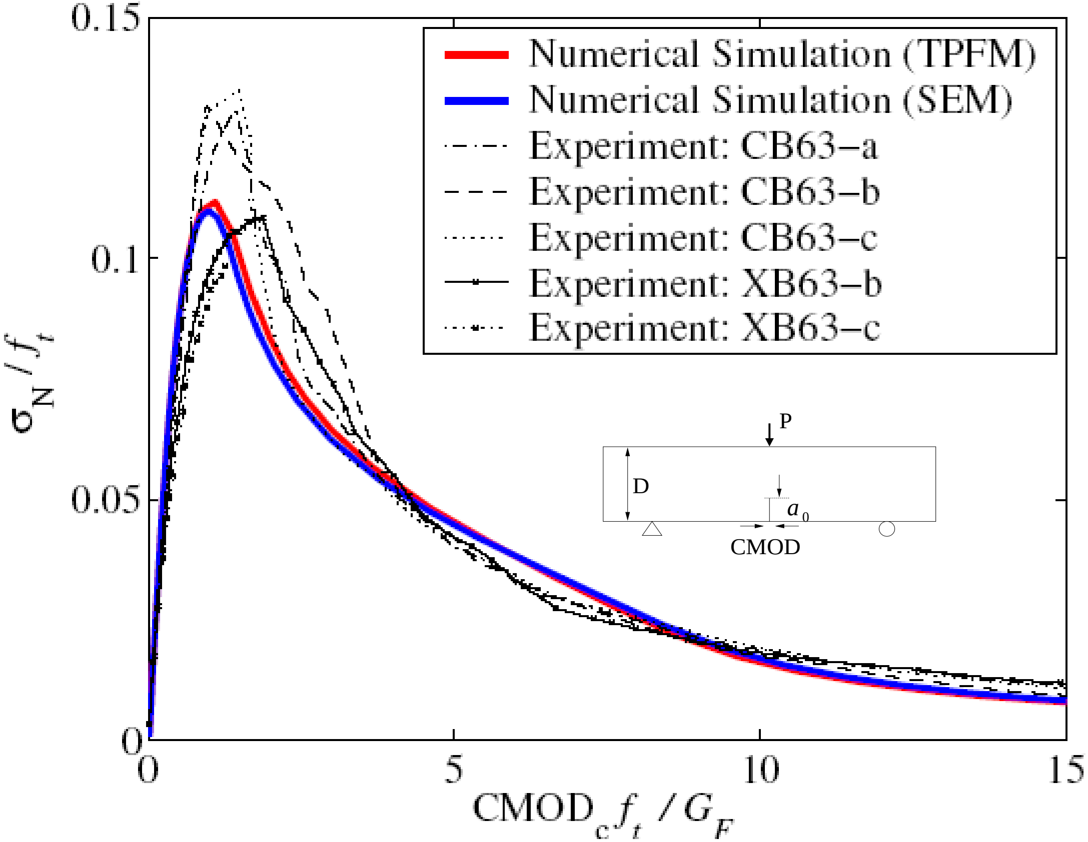
D = 63 (mm)

f_t' = 4.15 (MPa)

G_f = 56.6 & 52.1 (N/m)

G_F = 119 (N/m)

ψ = 0.25



Numerical Validation – Intermediate Beam

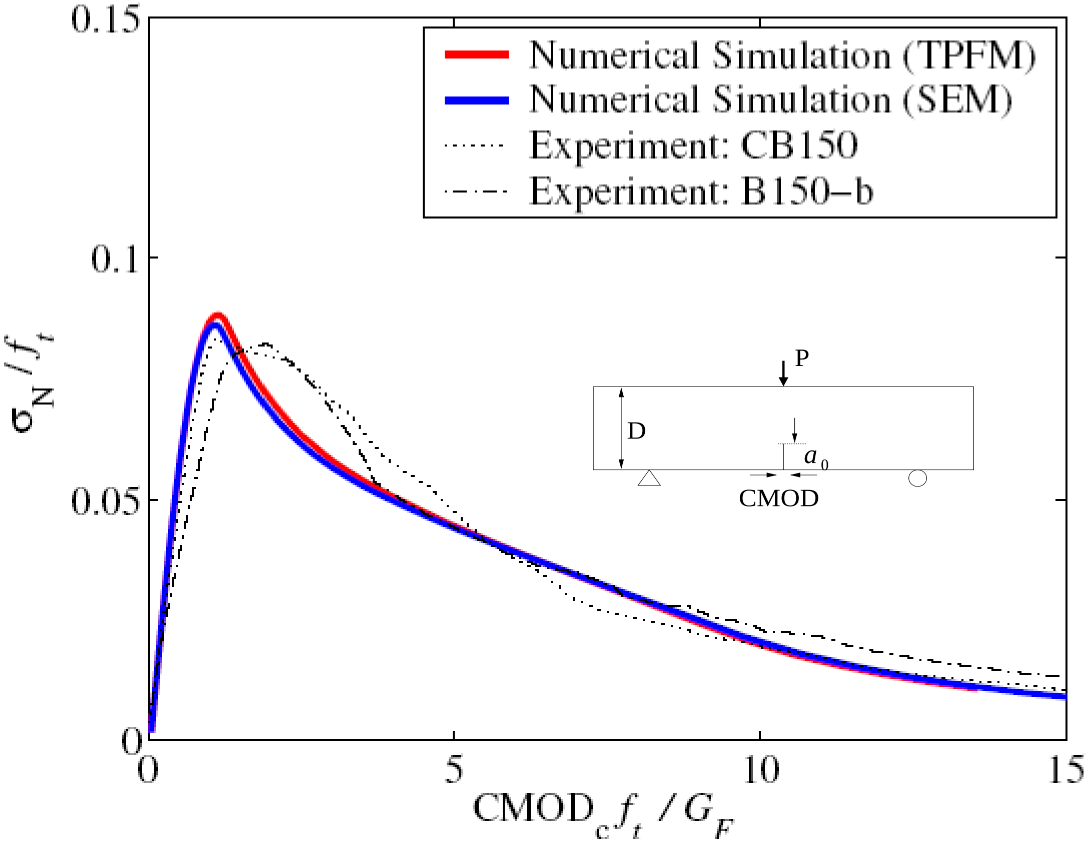
D = 150 (mm)

f_t' = 4.15 (MPa)

G_f = 56.6 & 52.1 (N/m)

G_F = 164 (N/m)

ψ = 0.25



Numerical Validation – Large Beam

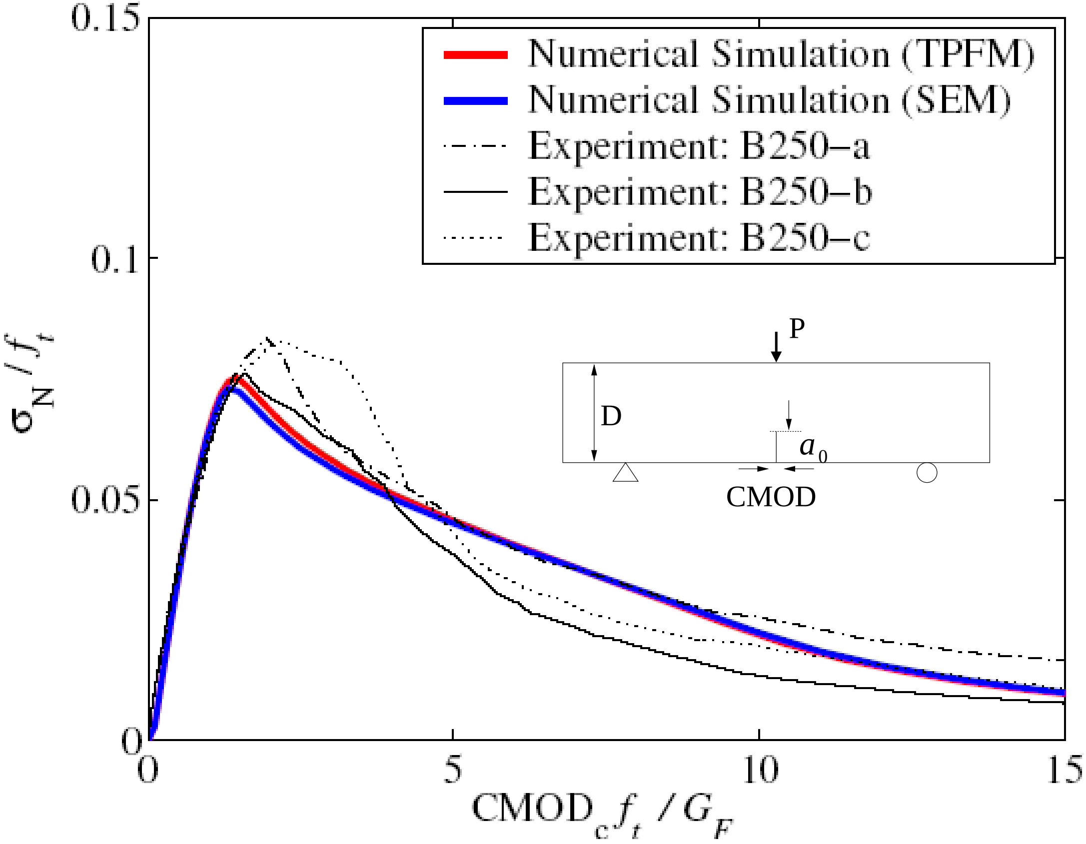
D = 250 (mm)

f_t' = 4.15 (MPa)

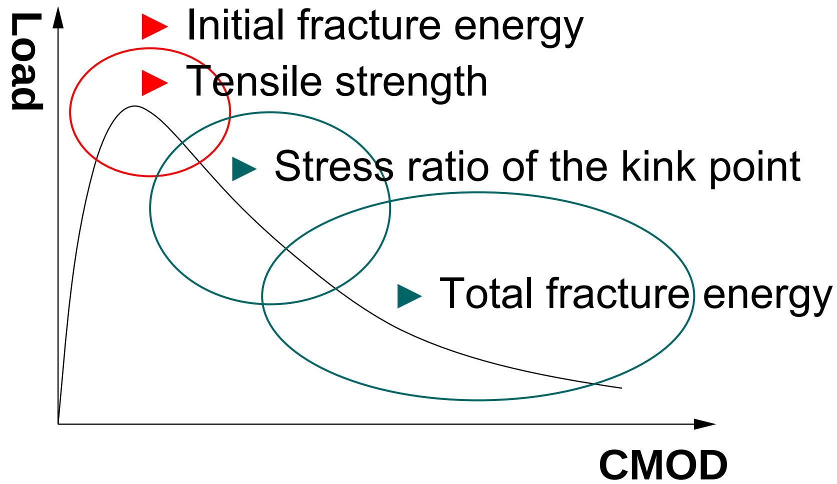
G_f = 56.6 & 52.1 (N/m)

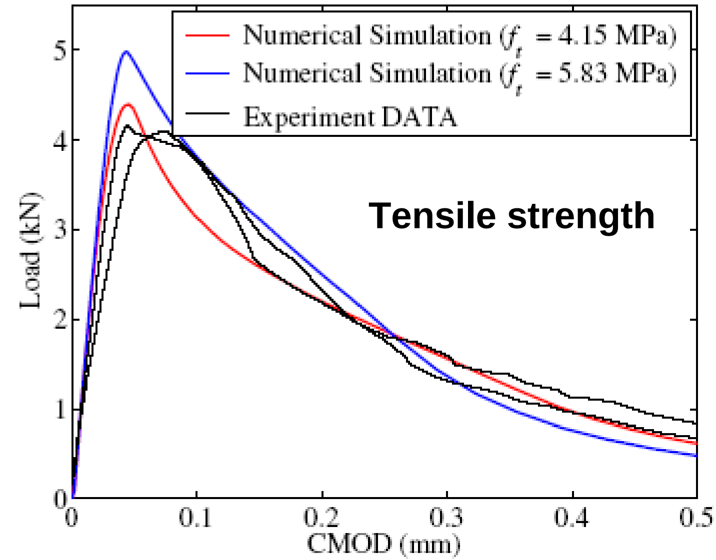
G_F = 167 (N/m)

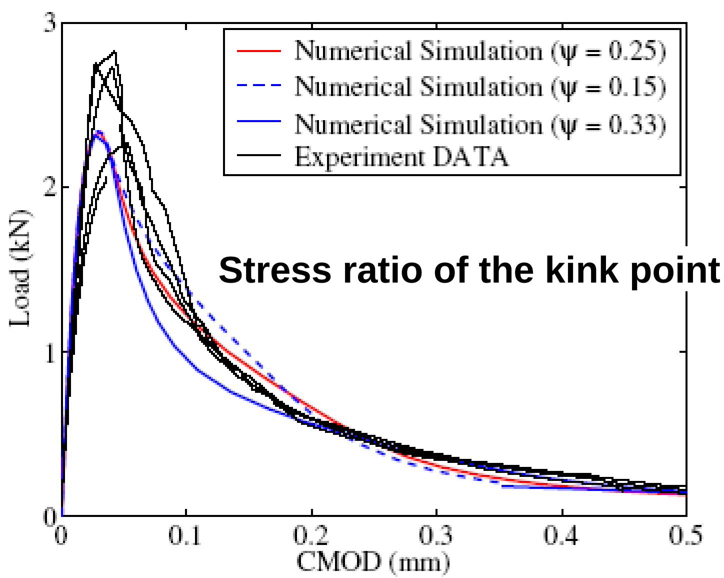
ψ = 0.25

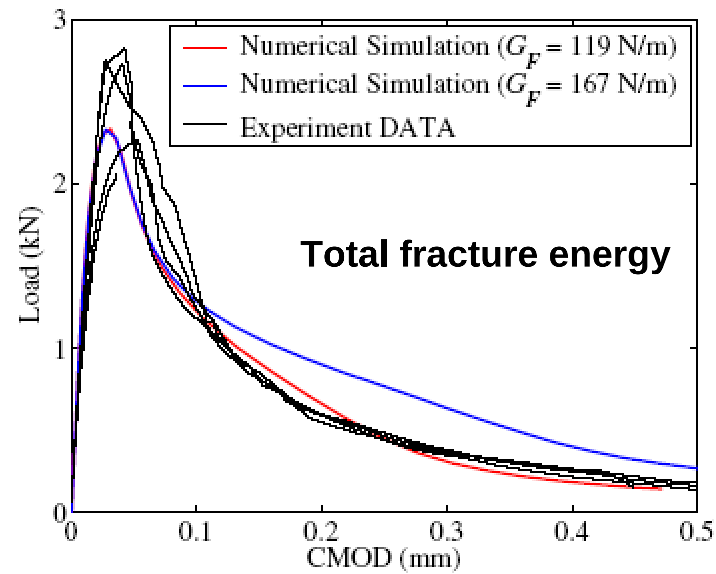


Model Sensitivity

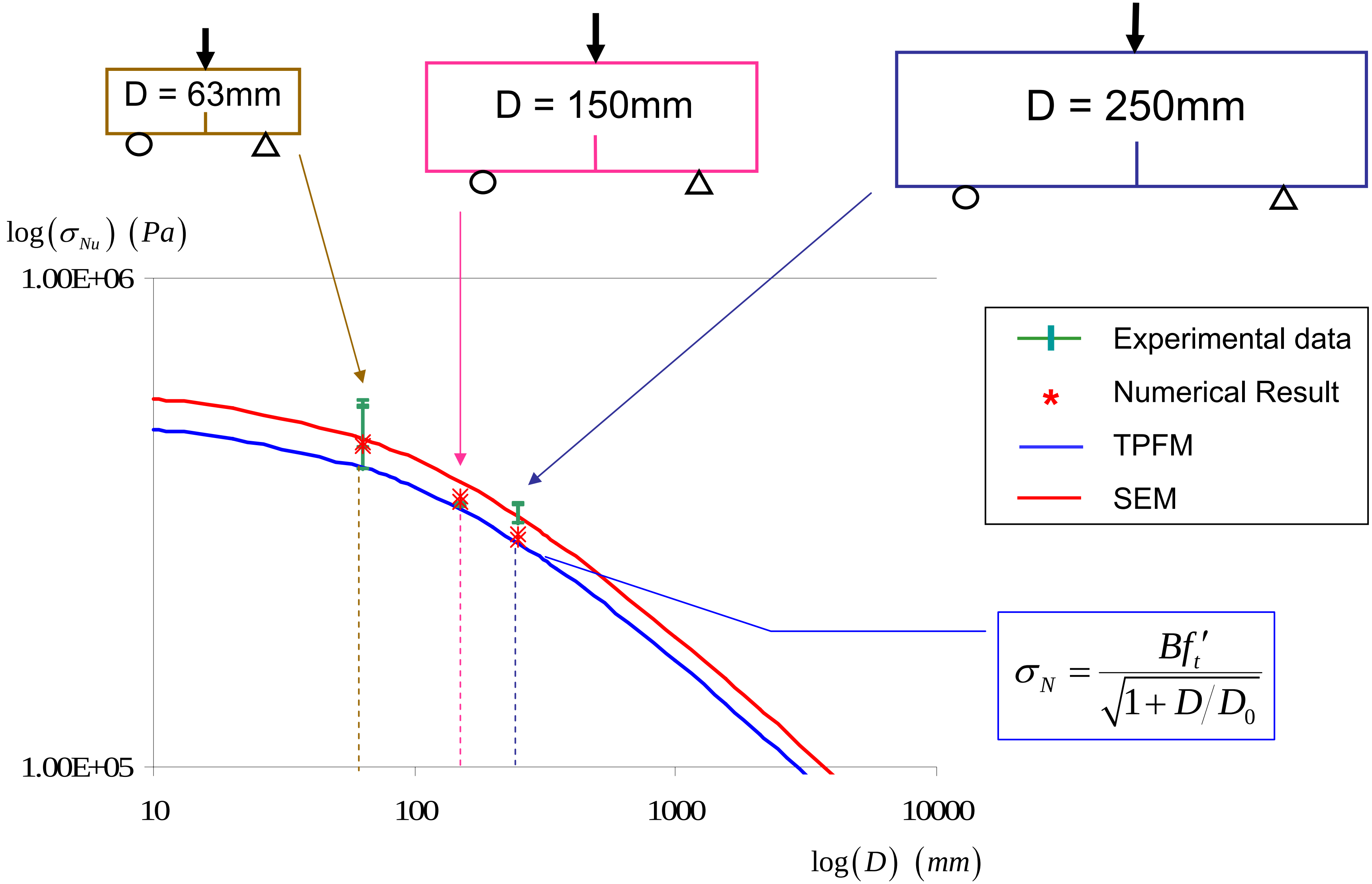








Size Effect


D = 63mm
D = 150mm
D = 250mm
log(σ_Nu) (Pa)
1.00E+06
1.00E+05
10
100
1000
10000
log(D) (mm)
Experimental data
Numerical Result
TPFM
SEM
$\sigma_N = \frac{Bf_t'}{\sqrt{1+D/D_0}}$

Summary

Predict Load-CMOD Curve

- Bi-linear softening cohesive zone model
- Without calibration of the fracture parameters.

Investigate Size Effect

- Cohesive Zone Model with bi-linear softening
- Experiment results from Three-point bending tests
- Size effect expression: $\sigma_N = \frac{Bf_t'}{\sqrt{1 + D/D_0}}$
- Good agreement between the results from the three methods.