System Reliability-Based Topology Optimization of Structures under Stochastic Excitations

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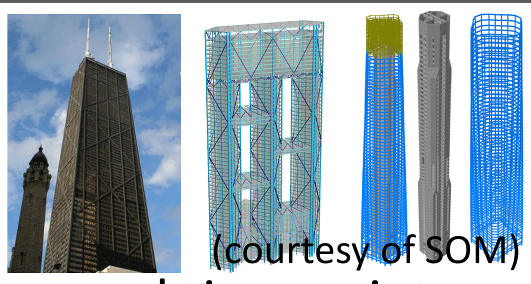
Research Objective

- Find optimal bracing systems under stochastic excitation, particularly induced by earthquake ground motions.
- Develop topology optimization framework Integrated with random vibration theory and structural reliability analysis.
- Evaluate the system-level failure probability accurately considering statistical dependence between failure modes, locations and time points.


(courtesy of SOM)

Discrete Representation Method

The discrete representation method discretizes a continuous stochastic process with a finite number of standard normal random variables.

Discretization of Random Process

$$f(t) = \mu(t) + \sum_{i=1}^{n} v_i s_i(t) = \mu(t) + \mathbf{s}(t)^{\mathrm{T}}\mathbf{v}$$

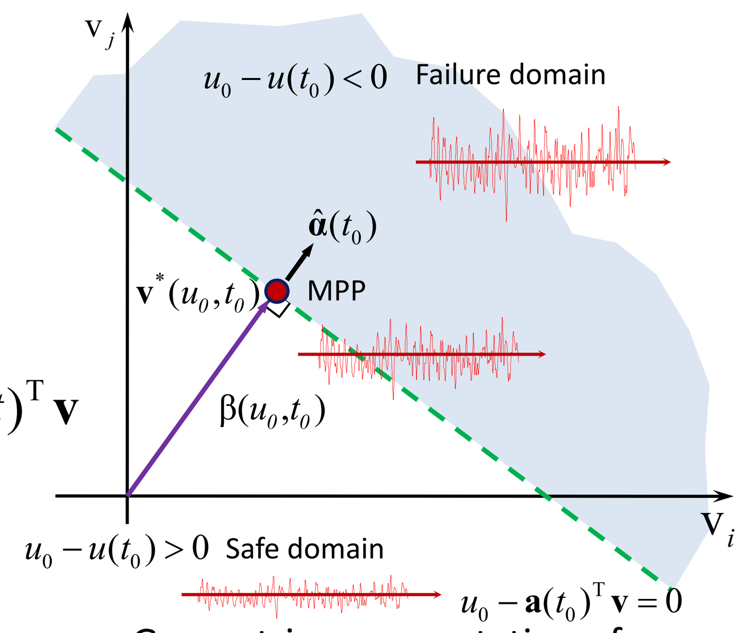
Stochastic Response

$$u(t) = \int_0^t \sum_{i=1}^{n} v_i s_i(\tau) h_s(t-\tau)\, d\tau = \sum_{i=1}^{n} v_i a_i(t) = \mathbf{a}(t)^{\mathrm{T}}\mathbf{v}$$

Instantaneous Failure Probability

$$P(E_f) = P(u_0 - \mathbf{a}^{\mathrm{T}}(t_0)\mathbf{v} \le 0) = P(g(u) \le 0)$$

$$g(u) = u_0 / \|\mathbf{a}(t_0)\| - (\mathbf{a}^{\mathrm{T}}(t_0)/\|\mathbf{a}(t_0)\|)\mathbf{v} = \beta(u_0, t_0) - \hat{\boldsymbol{\alpha}}(t_0)\cdot\mathbf{v}$$

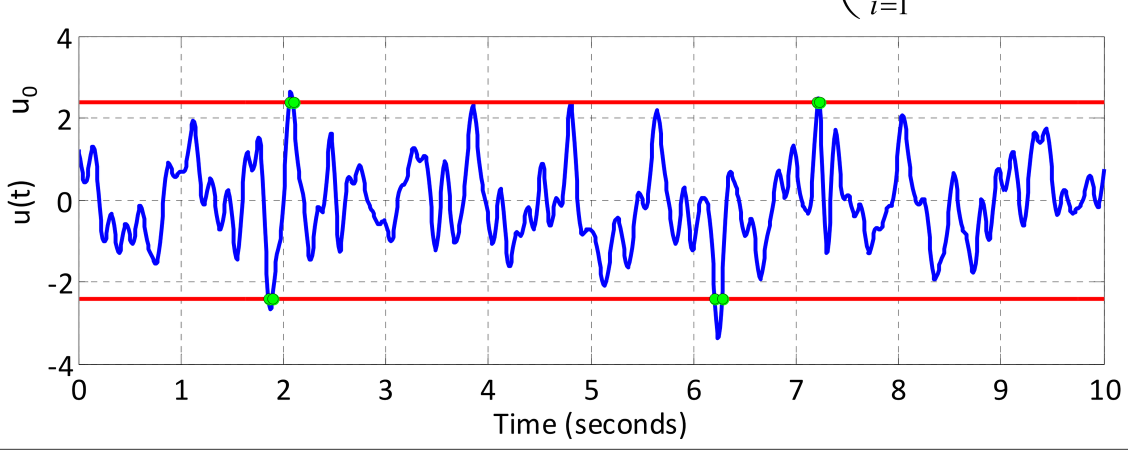


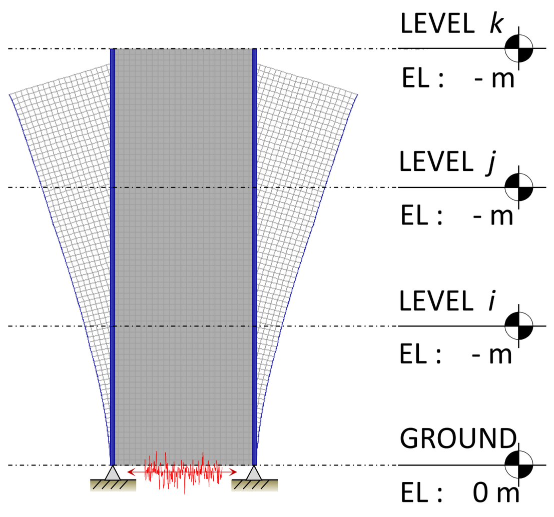
Geometric representation of instantaneous failure probability

The First Passage Probability

The probability that a stochastic response exceeds a given threshold at least once for a given duration. It is often used to describe the reliability of a system subjected to stochastic excitations.

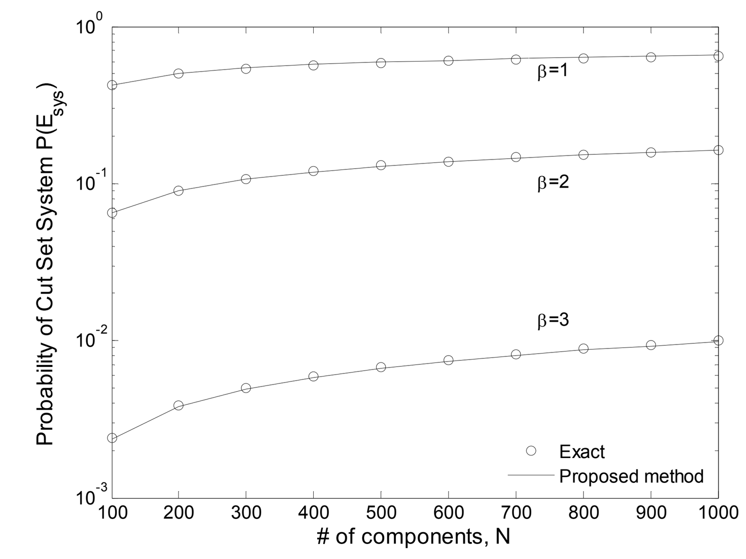
$$P(E_{sys}) = P(u_0 < \max_{0<t<t_n} |u(t)|) = P\left(\bigcup_{i=1}^{n} |u(t_i)| > u_0\right)$$





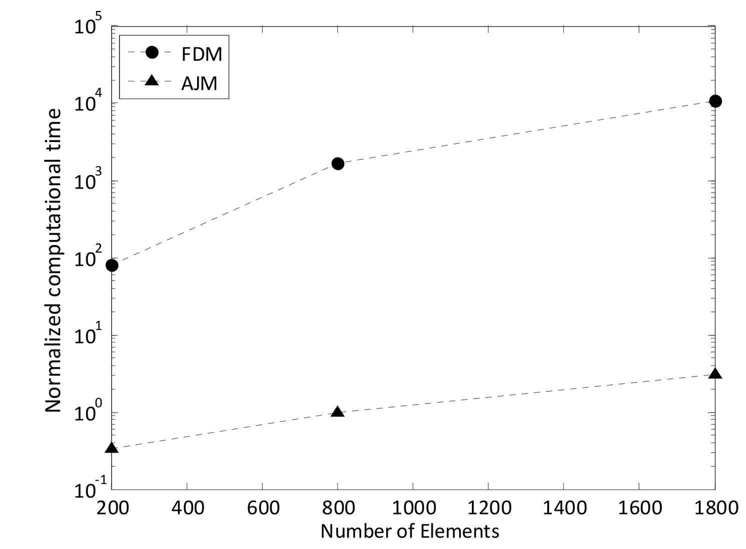
Sequential Compounding Method (SCM)

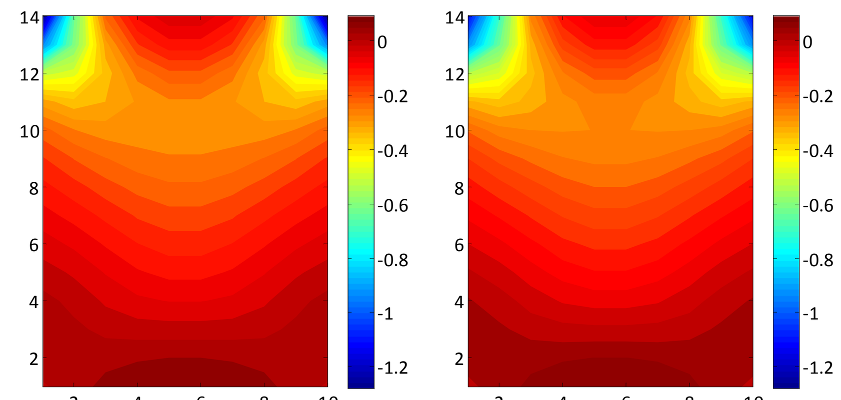
SCM can compute the probability of a large-system reliability problem efficiently and accurately.

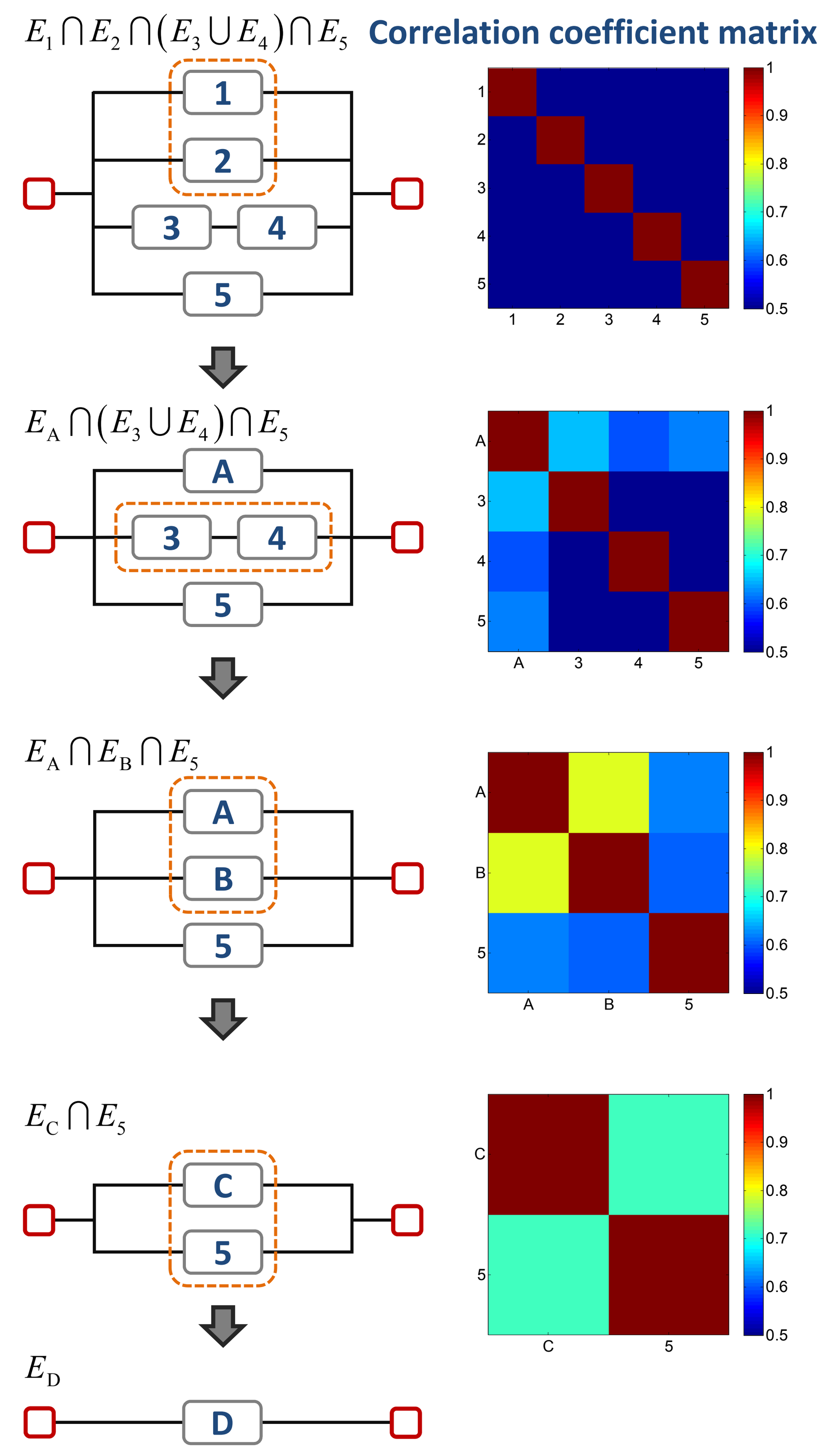


Performance of SCM

Sensitivity Analysis




Adjoint method | Finite difference method



Sequential compounding procedure

Lateral Load Resisting System Design

The purpose of topology optimization is to identify the optimal distribution of materials in a given design domain subjected to tractions and boundary conditions while satisfying given design constraints.

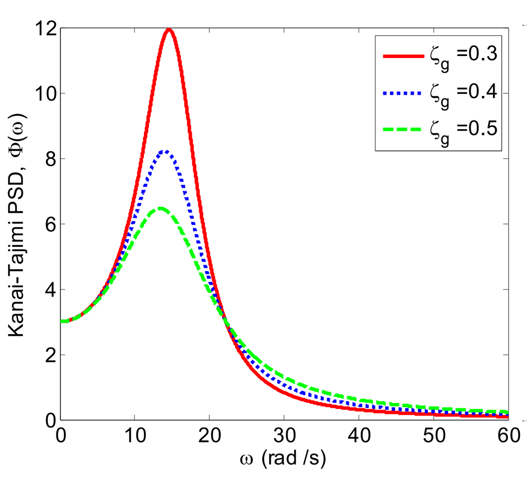
Stochastic Topology Optimization Framework

$$\min_{\mathbf{d}} \quad \text{Volume}(\tilde{\boldsymbol{\rho}}(\mathbf{d}))$$

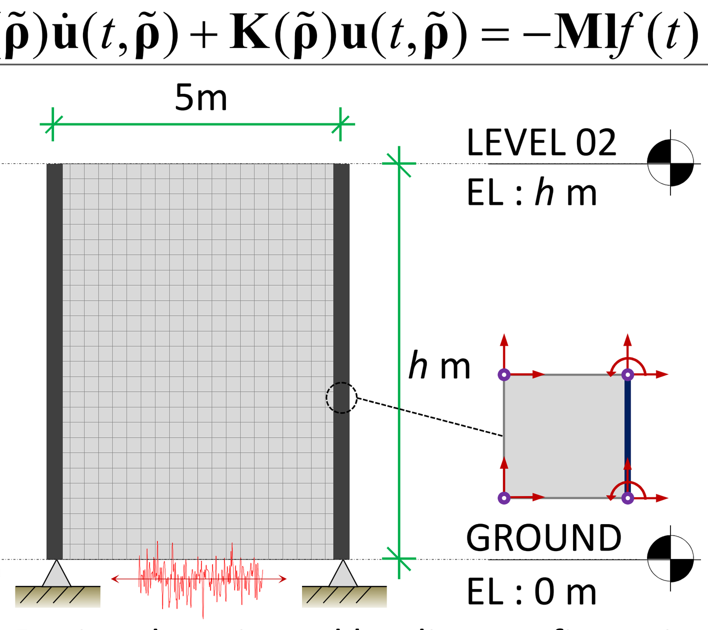
$$s.t \quad P\left(\bigcup_{i=1}^{70} |u(t_i)| > u_0\right) = P(E_{sys} : \beta_1, \ldots, \beta_{70}) \le P_{sys}^t (=2.3\%)$$

$$0 \le \tilde{\boldsymbol{\rho}}(\mathbf{d}, \mathbf{x}) \le 1 \quad \forall \mathbf{x} \in \Omega, \quad t_i \in [0,\ 3.5]$$

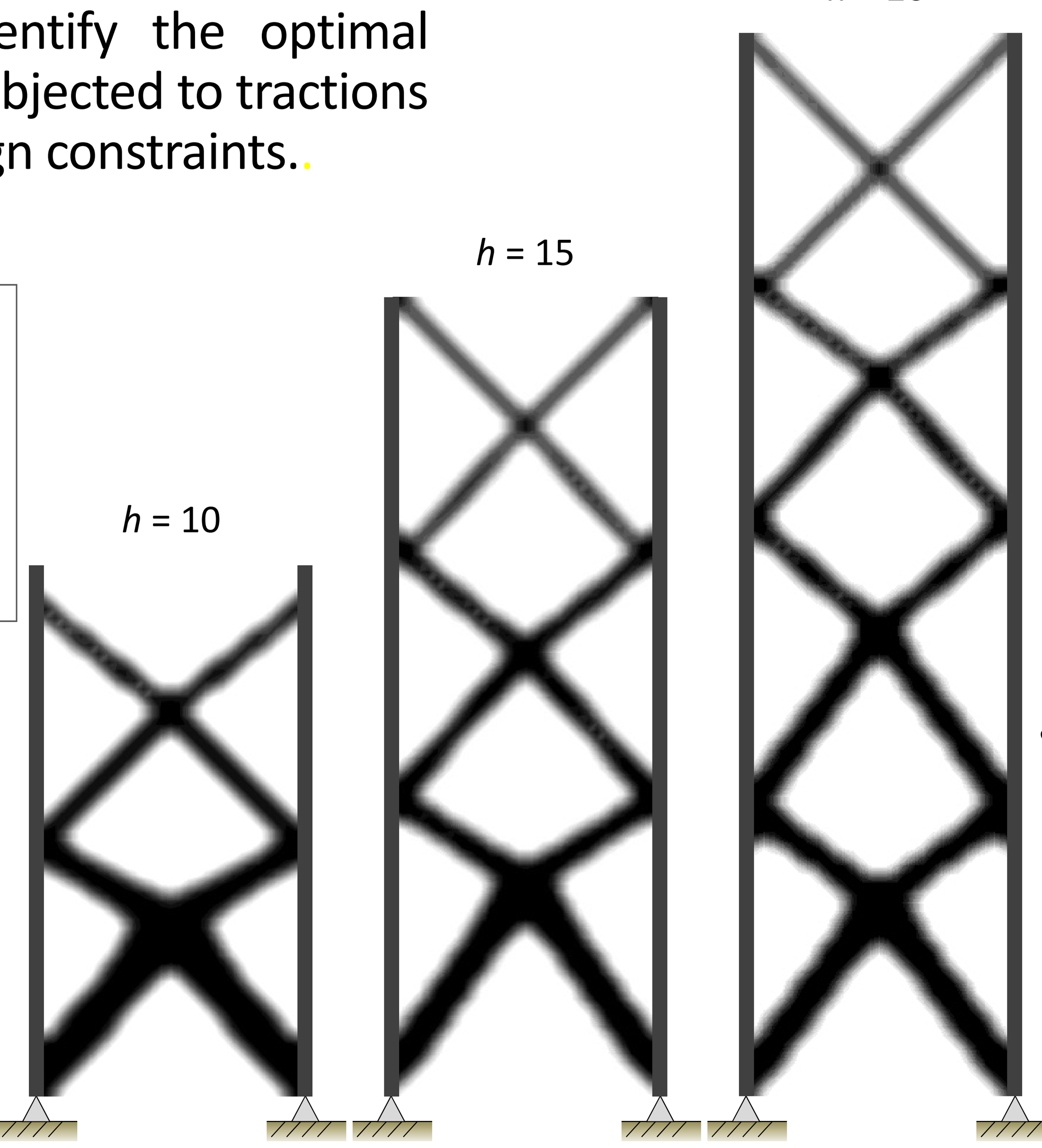
$$\text{with } \mathbf{M}(\tilde{\boldsymbol{\rho}})\ddot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{C}(\tilde{\boldsymbol{\rho}})\dot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{K}(\tilde{\boldsymbol{\rho}})\mathbf{u}(t,\tilde{\boldsymbol{\rho}}) = -\mathbf{M}\mathbf{l}f(t)$$



Kanai-Tajimi PSD

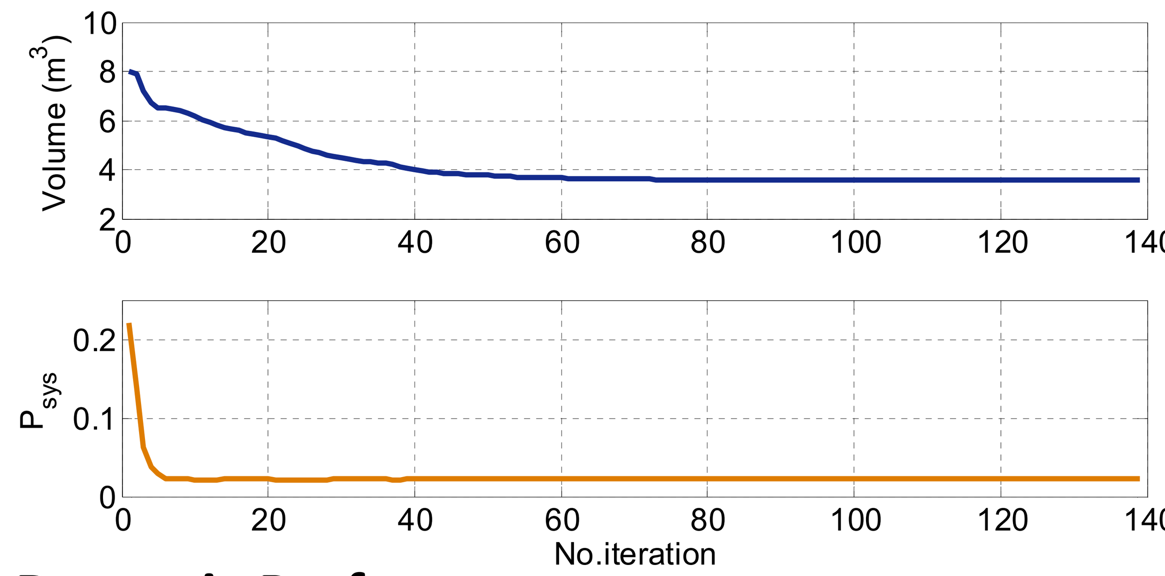


Design domain and loading configuration

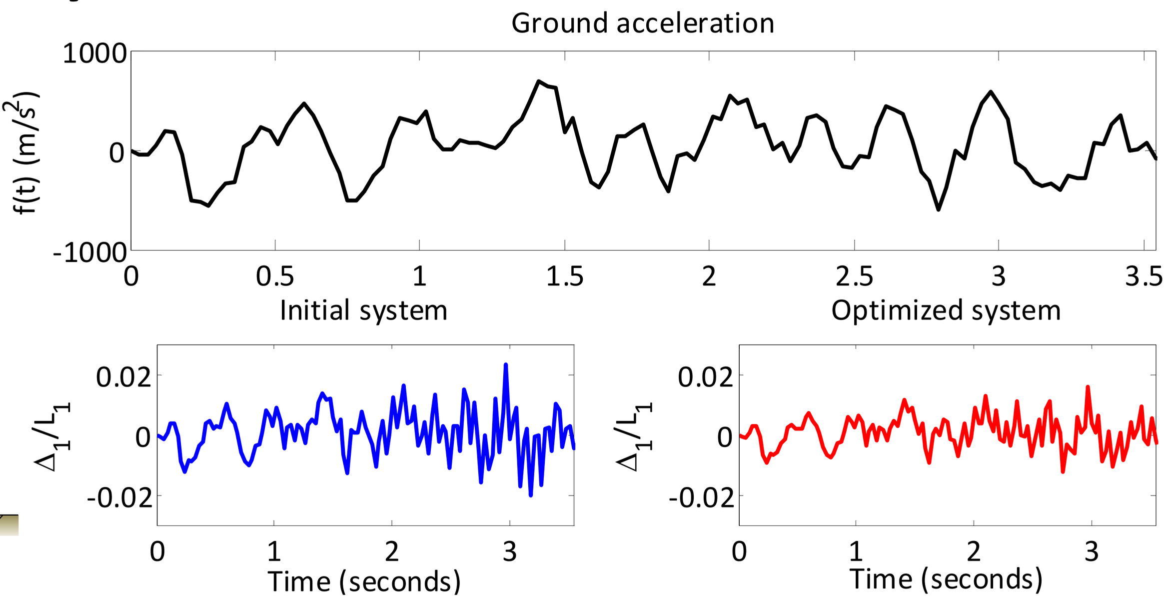


Φ_0=700 | Φ_0=30 | Φ_0=4

Convergence History



Dynamic Performance



Conclusion

- The sequential compounding method enables for an efficient and accurate computation of the failure probability of a large-size system reliability problem and its parameter sensitivities.
- The developed topology optimization framework under constraints on the first passage probability provides ways to find optimal bracing systems that can resist future realization of stochastic processes with a desired level of reliability.

Acknowledgements

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Reference

- J. Chun, J. Song and G. H. Paulino, Topology optimization of structures under stochastic excitations. *Computer Methods in Applied Mechanics and Engineering*, 2013. Submitted, under review.
- W.-H. Kang and J. Song, Evaluation of multivariate normal integrals for general systems by sequential compounding, *Structural Safety*, 32(1), 35-41, 2010.
- J. Song and A. Der Kiureghian, Joint first-passage probability and reliability of systems under stochastic excitation, *ASCE Journal of Engineering Mechanics*, 132(1), 65-77, 2006.
- A. Der Kiureghian, The geometry of random vibrations and solutions by FORM and SORM. *Probabilistic Engineering Mechanics*, 15(1), 81–90, 2000.