System Reliability-Based Topology Optimization under Stochastic Loads

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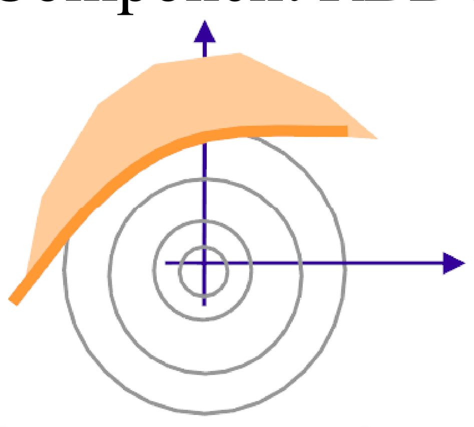
Research Objectives

- Account for the statistical dependence between the limit-states by using the matrix-based system reliability (MSR) method to compute the system failure probability and its sensitivities
- Employ a random vibration theory that predicts the dynamic response of a structure under random vibrations in stochastic manner in order to satisfy probabilistic constraints

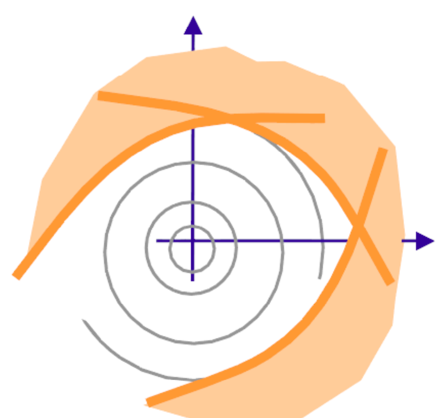
Reliability-Based Design Optimization (RBDO)

- Component RBDO



$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$
$$s.t.\quad P\; g_i(\mathbf{d},\mathbf{X})\le 0 \;\le P_i^t \quad i=1,\dots,n$$
$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U,\qquad \boldsymbol{\mu}_\mathbf{X}^L \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}_\mathbf{X}^U$$

- System RBDO



$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$
$$s.t.\quad P(E_{sys})=P\left[\bigcup_k \bigcap_{i\in C_k} g_i(\mathbf{d},\mathbf{X})\le 0\right]\le P_{sys}^t$$
$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U,\qquad \boldsymbol{\mu}_\mathbf{X}^L \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}_\mathbf{X}^U$$

Matrix-based System Reliability (MSR) Method

- To compute the probability of general system events in a uniform manner by use of simple matrix calculation

$$P_{sys} = \begin{cases} \int_\mathbf{s} \mathbf{c}^\mathrm{T}\mathbf{p}(\mathbf{s}) f_\mathbf{S}(\mathbf{s}) d\mathbf{s} \le P_{sys}^t & \text{dependent} \\ \mathbf{c}^\mathrm{T}\mathbf{p} \le P_{sys}^t & \text{independent} \end{cases}$$

c : event vector   p : probability vector

S : given an outcome of Common Source Random Variable(CSRV)

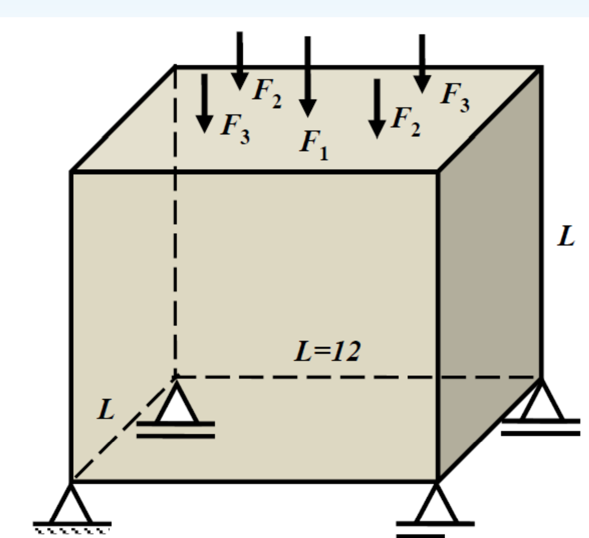
Parameter Sensitivity of System Reliability

- Statistically dependent components

$$P(E_{sys}) = \int_\mathbf{s} \mathbf{c}^\mathrm{T}\mathbf{p}(\mathbf{s}) f_\mathbf{S}(\mathbf{s}) d\mathbf{s}$$

$$\frac{\partial P_{sys}}{\partial\theta} = \int_\mathbf{s} \mathbf{c}^\mathrm{T}\left[\frac{\partial \mathbf{p}(\mathbf{s})}{\partial\theta} f_\mathbf{S}(\mathbf{s}) + \mathbf{p}(\mathbf{s})\frac{\partial f_\mathbf{S}(\mathbf{s})}{\partial\theta}\right] d\mathbf{s}$$

System Reliability-Based Topology Optimization



Objective: minimize volume $V(\rho)$

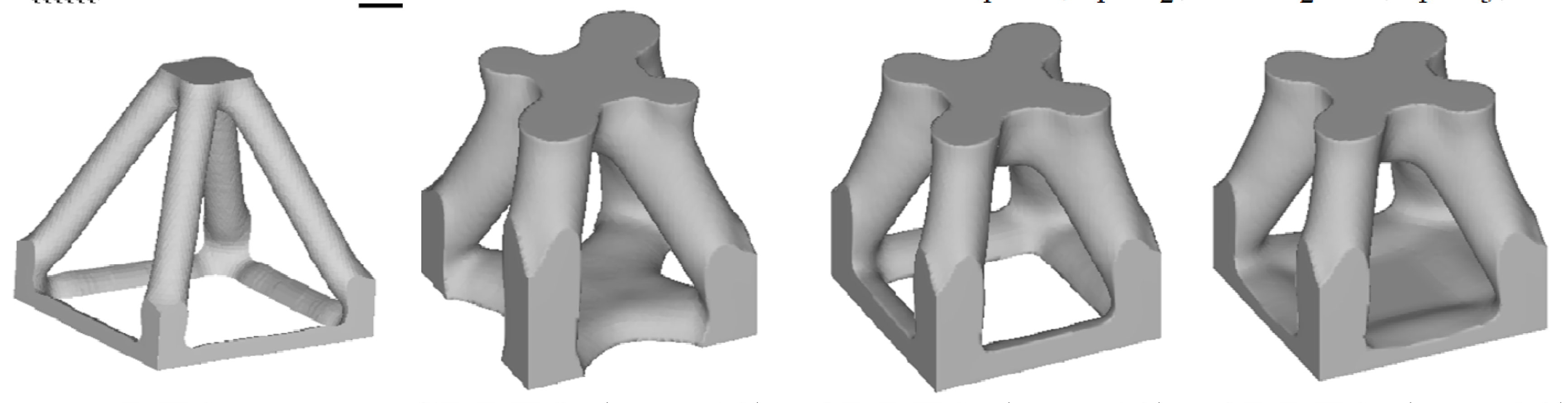
Limit-States : $C_i^t = 120$

$$g_i(\boldsymbol{\rho},\bar{\mathbf{F}}_i) = C_i^t - C_i(\boldsymbol{\rho},\bar{\mathbf{F}}_i) = C_i^t - \mathbf{u}^\mathrm{T}\mathbf{F}_i,\quad i=1,2$$

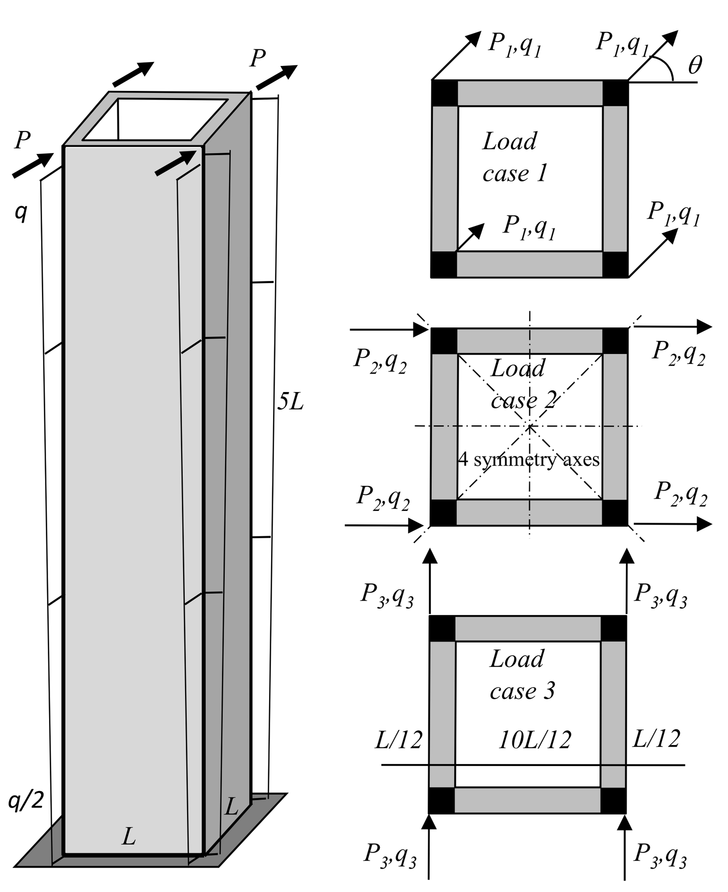
Random Loads :

F~(F_1,F_2,F_3)~N(100,10), N(0,30), N(0,40)

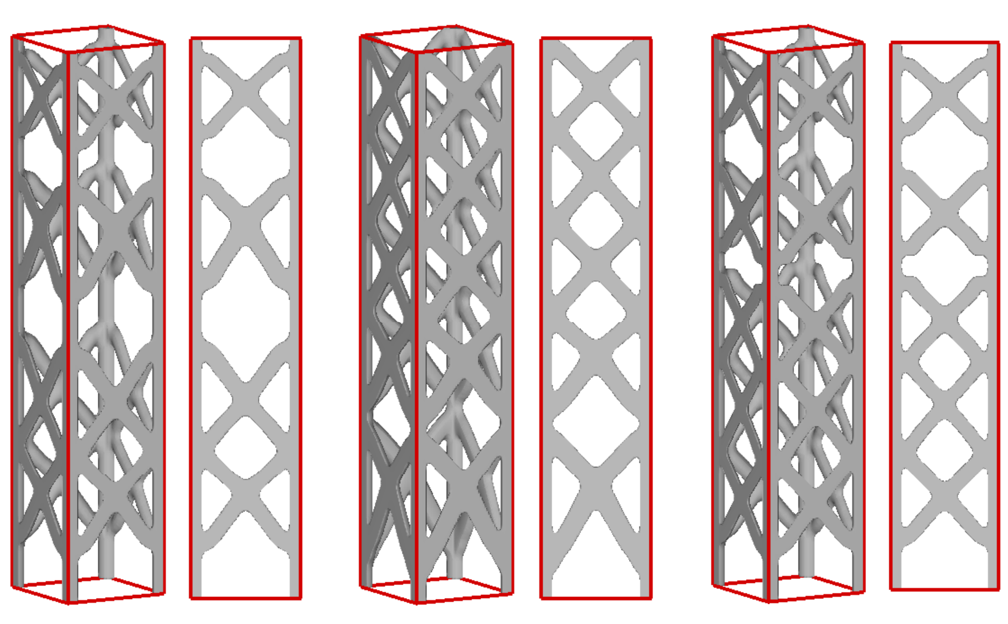
Load Cases : $\bar{\mathbf{F}}_1 = (F_1, F_2)$, $\bar{\mathbf{F}}_2 = (F_1, F_3)$



DTO *Volfrac = 6.3%* | **CRBTO** (σ_F =10) *Volfrac = 24.4%* | **SRBTO** (σ_F =10) *Volfrac = 22.3%* | **SRBTO** (σ_F =20) *Volfrac = 23.9%*



Courtesy of T.Nguyen



DTO *Volf= 21.93%* | **SRBTO** *Volf = 28.15%* $P_{sys}^t = 0.05$ | **SRBTO** *Volf = 22.25%* $P_{sys}^t = 0.85$

Conclusion and Future work

- SRBTO/MSR method is able to handle the statistical dependence between multiple limit-states
- Discrete representation method of a stochastic process will be employed to predict dynamic response of a structure in topology optimization

References

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- Song, J., and W.-H. Kang (2009). **System reliability and sensitivity under statistical dependence by matrix-based system reliability method.** Structural Safety, Vol. 31(2), 148-156.
- Der Kiureghian, A. (2000). **The geometry of random vibrations and solutions by FORM and SORM.** Probabilistic Engineering Mechanics, 15(1), 81-90.

Discrete Representation of a Stochastic Process

- Stochastic excitation can be discretized and represented in terms of a finite number of standard normal random variables

$$f(t) = \mu(t) + \sum_{i=1}^{n} u_i s_i(t) = \mu(t) + \mathbf{s}(t)^\mathrm{T}\mathbf{u}$$

Response of linear system to Gaussian excitation

$$x(t) = \int_0^t \sum_{i=1}^{n} u_i s_i(\tau) h(t-\tau)\,\mathrm{d}\tau = \sum_{i=1}^{n} u_i a_i(t) = \mathbf{a}(t)^\mathrm{T}\mathbf{u}$$

Failure Probability

$P[x(t_0) > x_0] = \Phi[-\beta(x_0, t_0)]$ x_0: threshold value

Example : Linear oscillator subjected to stochastic process

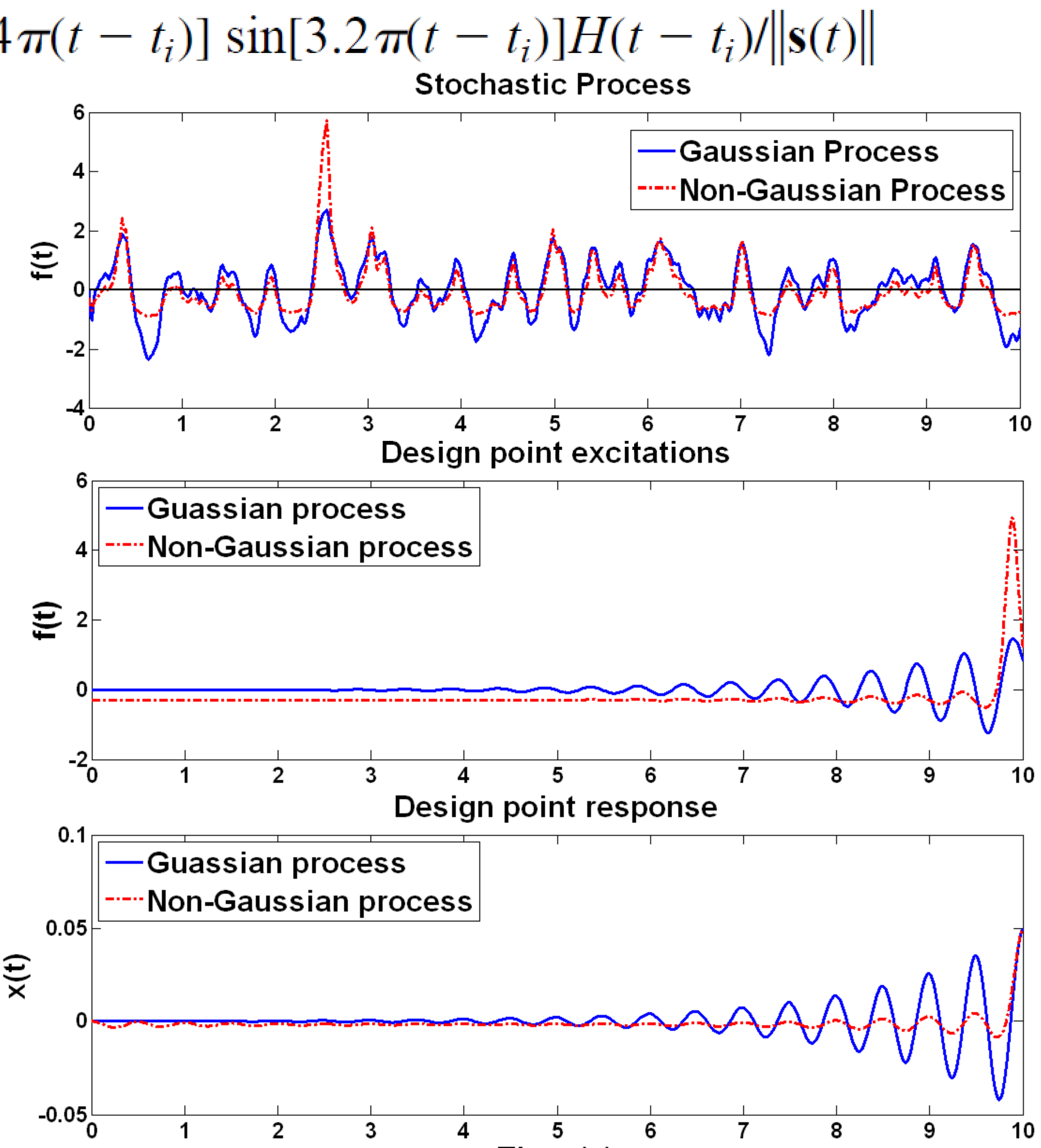
$$\ddot{x} + 2\zeta_0\omega_0\dot{x} + \omega_0^2 x = f(t)$$
$$s_i(t) = \exp[-2.4\pi(t-t_i)]\sin[3.2\pi(t-t_i)]H(t-t_i)/\|\mathbf{s}(t)\|$$

Threshold value, $x_0 = 0.05$

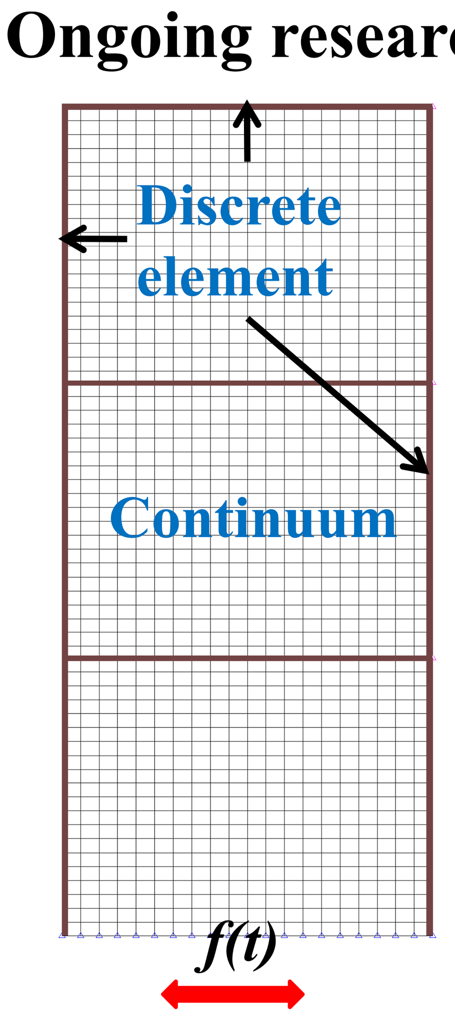
Time duration, $t_0 = 10$ *sec*

Reliability index, $\beta = 2.6929$

Failure probability $P_f = 0.0035$



Ongoing research



- **Topology optimization** under **probabilistic constraints**
- ***f(t)*** : earthquake excitation (discrete representation)