Finite Element Methods
(in Solid and Structural Mechanics)

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Prof. Glaucio H. Paulino
Donald Biggar Willett Professor of Engineering
Department of Civil and Environmental Engineering
University of Illinois at Urbana-Champaign

Acknowledgements:
J. Kim, Z. Zhang, S. Song, C. Le and K. Park
Acknowledgements

The slide series for this class has been developed with the help from the following people.

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<tr>
<th>J. Kim</th>
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Content

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- Course Outline (See Handout)
- Course Information (See Handout)
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- Text books
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Handouts

Paper Handouts

- Course outline
- Course information
- Course schedule

Electronic Handouts

- MATLAB
- Review of elasticity (CEE471)
- This ppt presentation

Note: These handouts are clipped together.
# CEE570/CSE551 Teaching Team

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<tr>
<th>Instructor</th>
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| Prof. Paulino  
Professor Willett Prof. of Eng.  
Faculty: CEE, CSE | Sofie Leon  
Ph.D. Student  
CEE | Tomas Zegard  
Ph.D. Student  
CEE | Junho Chun  
Ph.D. Student  
CEE | Heng Chi  
Ph.D. Student  
CEE | Xiaojia (Shelly) Zhang  
Ph.D. Student  
CEE |
This course is an introduction to basic continuum mechanics that emphasizes variational formulations and some basic theoretical/numerical aspects. It provides the prerequisite information for finite elements for graduate and advanced undergraduate students in engineering.
Pre-requisites for CEE570/CSE551: FEM

- **CEE 471 (Structural Mechanics)**
  - Tensors; geometry of deformation; constitutive relations; energy principles; boundary value problem; beam theory; plate theory; static stability theory; computational methods.

OR

- **TAM 551 (Solid Mechanics I)**
  - Mechanics of elastic deformable bodies, based on the fundamental concepts of modern continuum mechanics: kinematics, balance laws, constitutive equations; classical small-deformation theory; formulation of initial boundary-value problems of linear elastodynamics and boundary-value problems of linear elastostatics; variational formulations, minimum principles; applications of theory to engineering problems
Course Logistics

- **URL:**
  http://paulino.cee.illinois.edu/fem_login.html

- **Computer Labs.:**
  EWS: http://www.ews.uiuc.edu

- **Software Required (available at EWS)**
  - MATLAB
  - PATRAN
  - ABAQUS
Class Description

- Theory and applications of the finite element method;
- General procedure for solving a structural problem using FEM;
- Variational Methods; Weighted Residual (WR) Methods
- Review of one-dimensional elements;
- Techniques for constructing element stiffness matrices;
- Various two- and three-dimensional solid elements;
- Isoparametric formulation;
- Plates and shells elements;
- Stress recovery and post processing;
- Other topics: error, convergence, solving linear system of equations...

Textbook

1. Introduction
2. One-Dimensional Elements and Computational Procedures
3. Basic Elements
4. Formulation Techniques: Variational Methods
5. Formulation Techniques: Galerkin and Other Weighted Residual Methods
6. Isoparametric Elements
7. Isoparametric Triangles and Tetrahedra
8. Coordinate Transformation and Selected Analysis Options
9. Error, Error Estimation, and Convergence
10. Modeling Consideration and Software Use
11. Finite Elements in Structural Dynamics and Vibrations
12. Heat Transfer and Selected Fluid Problems
13. Constraints: Penalty Forms, Locking, and Constraint Counting
14. Solids of Revolution
15. Plate Bending
16. Shells
17. Nonlinearity: An Introduction
18. Stress Stiffness and Buckling

**Practical Application**

**Very Important**

**Theoretical**


Our book has something for everyone …
Recommended Textbooks

K.-J. Bathe (1995)  
*Finite Element Procedures* Prentice Hall

R. D. Cook (1995)  
*Finite Element Modeling for Stress Analysis* John Wiley and Sons
Recommended Textbooks

T. J. Hughes (2000)
*The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*
Dover Publications

J. N. Reddy (2005)
*An Introduction to the Finite Element Method*

*An Introduction to Nonlinear Finite Element Analysis*
Oxford University Publication
Recommended Textbooks

M. Asghar Bhatti (2005)

*Fundamental Finite Element Analysis and Applications: with Mathematica and Matlab Computations*

John Wiley and Sons
Recommended Textbooks


*The Finite Element Method*

4th Edition, John Wiley and Sons

Recommended Textbooks

O. C. Zienkiewicz, et. al (2005)

*The Finite Element Method*


Vol 1: *The Basis*
Vol 2: *Solid Mechanics*
Vol 3: *Fluid Mechanics*
We want to know the states (displacement, strains, stresses...) throughout the structure.
Governing relations to be satisfied:

1. Constitutive relations: relation between stresses and strains (determined by material properties)

2. Kinematic relations: relation between displacement and strains (usually in the form of differential equations)

3. Equilibrium of forces (usually in the weak form of the Principle of Virtual Work, the Principle of Stationary Potential Energy, or the vanishing of weighted residuals)

Closed form solution is not possible for a general case. Approximate numerical solution is employed: the FEA.
Simulation process (broad view)

- Physical system
  - IDEALIZATION
  - Beam theory
  - Plate theory
  - ...

- Mathematical system
  - Finite element methods...

- Discrete Model
  - Model error
  - Discretization + solution error
  - e.g. iterative refinement

- Discrete Solution
  - Solution error
  - e.g. adaptive remeshing

- Change models
  - (e.g. beam ↔ solid)

The above flowchart is based on the one given in Felippa’s book.
Finite Element Analysis Procedure

**Discretization**
(divide the structure into small, simple elements)

**Localization**
(obtain the behavior of each element)

**Globalization (Assembly)**
(relate all elements based on the connectivity)

**Solution and post processing**
(solve for state variables and recover quantities of interest, such as stress)

\[ K^e u^e = f^e \]

\[ Ku = f \]
Next Class

- Introduction to FEM
- History of FEM
- Advantages and shortcomings of FEM
- Misuses of FEA programs