Problem 1. LOCAL-GLOBAL TRANSFORMATIONS – BEAM ELEMENT
Consider the two-dimensional (2D) beam element shown below:

\[
\begin{align*}
\cos(\theta) &= \frac{x_2 - x_1}{L} \\
\sin(\theta) &= \frac{y_2 - y_1}{L} \\
L &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\end{align*}
\]

where \(\{\delta\}\) is the member end displacement vector in global directions,

\[\{\delta\} = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6\}^T\]

and \(\{\delta'\}\) is the member end displacement vector in local directions,

\[\{\delta'\} = \{\delta'_1, \delta'_2, \delta'_3, \delta'_4, \delta'_5, \delta'_6\}^T\].

- **Verify** that the relationship between \(\{\delta'\}\) and \(\{\delta\}\) is

\[
\{\delta\}_{6\times1} = [T]_{6\times6}\{\delta'\}_{6\times1}
\]

where

\[
[T] = \begin{bmatrix}
c & -s & 0 & 0 & 0 & 0 \\
s & c & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & c & -s & 0 \\
0 & 0 & 0 & s & c & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

and

\[c = \cos(\theta) \quad , \quad s = \sin(\theta)\]
The member stiffness matrix in local coordinates is

\[
[K'] = \begin{bmatrix}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\tag{3}
\]

where \(A\) is the cross-sectional area, \(I\) is the moment of inertia, \(E\) is the Young’s modulus, and \(L\) is the length of the member.

- Use the congruent transformation (see Laboratory #2, Problem 1)

\[
[K] = [T][K'][T]^T
\]

to find the global stiffness matrix \([K]\), i.e. perform the matrix multiplications above by yourself (no computer).

- Write a MATLAB program (i.e. function) to calculate the global stiffness matrix \([K]\) (output) once you input \(E\), \(A\), \(L\), \(I\), and \(\theta\) (use the matrix that you obtained in the previous item).

- Using your MATLAB program (developed in the previous item), calculate \([K]\) for members #2 and #3 of the gable frame below.

- Check the answer that you obtained in the previous item by using the MATLAB program that you wrote for Laboratory #2, Problem 1.

![Diagram of gable frame with member properties]

**Member Properties**

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| 1 & 4 | \(A = 40 \text{ in}^2\)  
| I = 4000 \text{ in}^2 |
| 2 & 3 | \(A = 30 \text{ in}^2\)  
| I = 6000 \text{ in}^2 |

For all members, \(E = 30,000 \text{ ksi}\)
Problem 2. PRACTICE PROBLEMS WITH MATLAB

1. **Equation of a straight line:** The equation of a straight line is \( y = mx + c \) where \( m \) and \( c \) are constants. Compute the \( y \)-coordinates of a line with slope \( m = 0.4 \) and the intercept \( c = -2.5 \) at the following \( x \)-coordinates:

\[
x = 0, 1.5, 3, 4, 5, 7, 9, \text{ and } 10
\]

**NOTE:** Your command should NOT involve any array operators since your calculation involves multiplication of a vector with a scalar \( m \) and then addition of another scalar \( c \).

2. **Points on a circle:**

All points with coordinates \( x = r \cos \theta \) and \( y = r \sin \theta \) where \( r \) is a constant, lie on a circle with radius \( r \), i.e. they satisfy the equation \( x^2 + y^2 = r^2 \). Create a column vector for \( \theta \) with the values \( 0, \pi/4, \pi/2, 3\pi/4, \pi, \text{ and } 5\pi/4 \). Take \( r = 5 \) and compute the column vectors \( x \) and \( y \). Now check that \( x \) and \( y \) indeed satisfy the equation of circle, by computing the radius \( r = \sqrt{x^2 + y^2} \).

**NOTE:** To calculate \( r \) you will need the array operator \( \cdot \) for squaring \( x \) and \( y \). Of course, you could compute \( x^2 \) by \( x.*x \) also.

3. **The geometric series:**

**NOTE:** Now you know how to compute \( x^n \) element by element for a vector \( x \) and a scalar exponent \( n \). How about computing \( n^x \), and what does it mean? The result is again a vector with elements \( n^{x_1}, n^{x_2}, n^{x_3}, \text{ etc} \).

The sum of a geometric series \( 1 + r + r^2 + r^3 + \cdots + r^n \) approaches the limit \( \frac{1}{1-r} \) for \( r < 1 \) as \( n \to \infty \). Create a vector \( x = [r^0, r^1, r^2, \cdots, r^n] \) with the command \( x=r.^n \).

Now take the sum of this vector with the command \( s = \text{sum}(x) \) (\( s \) is the sum of the actual series). Calculate the limit \( \frac{1}{1-r} \) and compare with the computed sum \( s \). Repeat the procedure by taking \( n \) from 0 to 50 and then from 0 to 100.

4. **Compute geometrical properties of a primitive shape:**

Consider a rectangle with basis \( b \) and height \( h \). The datum (point from which the dimensions to the centroid are measured) is located in the lower left corner. Write a MATLAB function called "myrectangle" which receives as input \( b \) and \( h \) and outputs the area \( (A = bh) \), the perimeter \( (\text{circ} = 2(b + h)) \), the moment of inertia about the centroidal axis \( x \) \( (I_x = bh^3/12) \), the moment of inertia about the centroidal axis \( y \) \( (I_y = hb^3/12) \), the distance from datum to centroid in the \( x \) direction \( (\text{centx} = b/2) \), and the distance from datum to centroid in the \( y \) direction \( (\text{centy} = h/2) \).