MEMBER END-FORCES DUE TO MEMBER LOADS CONSIDERING RESTRAINED JOINTS

2) PLANE FRAME MEMBERS

a) APPLIED LOAD
   See Notes of yesterday (9/19/2000)

b) UNIFORM RISE OF TEMPERATURE: see Section 5.3.2.1 of textbook, page 125

\[ F = \kappa T A E \quad \text{(independent of length)} \]

\[ \{f\}' = \begin{bmatrix} F & 0 & 0 & -F & 0 & 0 \end{bmatrix}^T \]

Not considered for F-H

c) NON-UNIFORM RISE OF TEMPERATURE: see Section 5.3.2.2 of textbook, page 126
Compatibility Equation: \( \delta_{10} + R \delta_{11} = 0 \)

\[
\delta_{10} = \int_0^L \frac{M_1}{EI} \, dx = \frac{1}{E} \int_0^L \frac{a \Delta T}{h} \, ds = \frac{a \Delta T L}{h}
\]

same signs

\[
\delta_{11} = \int_0^L \frac{M_1 M_1}{EI} \, ds = \int_0^L \frac{(-1)^2}{EI} \, ds = \frac{L}{EI}
\]

Thus: \( \frac{a \Delta T}{h} L + R \frac{L}{EI} = 0 \) \( \Rightarrow \)

\[
R = \frac{-a \Delta T}{h} \frac{EI}{L}
\]

Interpretation:

Effect of Supports

w/o supports, beam tends to deform as shown
F - F:

\[
\begin{align*}
\{ f \} &= \{ F \circ R \circ F \}^T \\
F &= \alpha \tan \theta \frac{AE}{R} \quad \text{(as before)} \\
R &= \frac{\alpha \Delta \theta EI}{h}
\end{align*}
\]

What about \( F - H \)?

Can use similar procedure.

See Handout on "Method of Consistent Deformations".

Compatibility Equation:

\[
\delta_{10} + R \delta_{11} = 0 \\
\text{(same as before)}
\]

Recall:

\[
\delta_{11} = \frac{\alpha \Delta \theta ds}{h}
\]

\[
\begin{align*}
\delta_{10} &= \int_0^L M_1 \, d\theta = \int_0^L \left( \frac{x}{L} \right) \frac{\alpha \Delta \theta}{h} \, dx = \frac{\alpha \Delta \theta}{hL} \int_0^L x \, dx = \frac{\alpha \Delta \theta L^2}{2h} \\
\delta_{11} &= \int_0^L \frac{M_1^2}{EI} \, ds = \frac{1}{EI} \int_0^L \left( \frac{x}{L} \right)^2 \, dx = \frac{1}{EI} \frac{L}{3} \frac{L^3}{3} = \frac{L}{3EI}
\end{align*}
\]

Thus:

\[
\frac{\alpha \Delta \theta L^2}{2h} + \frac{R L}{3EI} = 0 \\
\Rightarrow \quad R_{FH} = -\frac{3}{2} \frac{\alpha \Delta \theta EI}{h}
\]
Quicker Way to handle the F-H case!

Thus:
\[
\frac{R_{F-H}}{R_{F-F}} = \frac{3}{2}
\]

Summary

**F-F:**
\[
F = \alpha T_c A e, \quad R = \frac{\Delta T E I}{h}
\]
\[
\{f\}' = \begin{bmatrix} F_0 & -R \end{bmatrix}^T
\]

**F-H:**
\[
F = \alpha T_c A e, \quad R = \frac{3}{2} \times \Delta T E I
\]
\[
\{f\}' = \begin{bmatrix} F_0 & -\frac{3}{2} R \end{bmatrix}^T
\]

Recall + convention:

The + convention is NOT important.
d) **FORCED DEFORMATIONS**

d.1) **SUPPORT ROTATION**

\[ F = \frac{3R}{2L} \]

Please see handout on "METHOD OF CONSISTENT DEFORMATIONS"

\[
\alpha \cdot L = \frac{1}{EI} \int_0^L M m \, dx = \frac{R L}{EI} \quad \Rightarrow \quad R = \frac{4EI}{L} \alpha
\]

See handout on INTEGRATION TABLE (single page) (Can use the 1st. case)

Thus:

\[
\{ f' \}_{F_F} = \begin{bmatrix} 0 & F & R \end{bmatrix}^T \quad R = \frac{4EI}{L} \alpha
\]

\[
F = \frac{3R}{2L} = \frac{6EI}{L^2} \alpha
\]
What about F-H case?
Use superposition

\[\{\mathbf{f}'\}_{F-H} = \{0, F^*, R^*, 0 - F^*, 0\}^T\]

Important
\[R^* = R - \frac{R}{4} = \frac{3R}{4} = \frac{3}{4} \frac{4EI}{L} \alpha = \frac{3EI}{L} \alpha\]
\[F^* = \frac{R^*}{L} = \frac{3}{4L} = \frac{3EI}{L^2} \alpha\]

Summary

\[\{\mathbf{f}'\}_{F-F} = \{0, F, R, 0, 0 - F, R/2\}^T\]
\[R = \frac{4EI}{L} \alpha, \quad F = \frac{6EI}{L^2} \alpha\]
\[\{\mathbf{f}'\}_{F-H} = \{0, F^*, R^*, 0 - F^*, 0\}^T\]
\[R^* = \frac{3EI}{L} \alpha, \quad F^* = \frac{3EI}{L^2} \alpha\]
d. 2) SUPPORT SETTLEMENT

Please see handout on "METHOD OF CONSISTENT DEFORMATIONS"

\[
\Delta \frac{2}{L} = \frac{1}{EI} \int_0^L M m \, dx = \frac{1}{EI} \frac{R L}{3} \quad \Rightarrow \quad R = \frac{6EI}{L^2} \Delta
\]

See handout on INTEGRATION TABLE (single page)
(can use 1st. case)

Thus,

\[
\{ f' \}_F = \begin{bmatrix} 0 & F & R & 0 \end{bmatrix}^T \quad R = \frac{6EI}{L^2} \Delta
\]

\[
F = \frac{2R}{L} = \frac{12EI}{L^3} \Delta
\]
What about F-H case?
Again, use superposition.

\[ R_1 \quad \oplus \quad R_2 \]
\[ \downarrow \]

\[ R^* \quad L \quad F^* \]

\[ \{ f' \} = \begin{bmatrix} 0 & F^* & R^* & 0 & -F^* & 0 \end{bmatrix}^T \]

**NOT IMPORTANT**

\[ R^* = R - \frac{R}{2} = \frac{R}{2} = \frac{1}{2} \frac{6EI}{L^2} = \frac{3EI}{L^2} \]

\[ F^* = \frac{R^*}{L} = \frac{3EI}{L^3} \Delta \]

**Summary**

\[ F-F: \]
\[ \Delta \]
\[ R \]
\[ \frac{\Delta}{R} \]
\[ F \]
\[ \frac{F}{L} \]

\[ \{ f' \}_{F-F} = \begin{bmatrix} 0 & F & R \end{bmatrix} \]

\[ R = \frac{6EI}{L^2} \Delta, \quad F = \frac{3EI}{L^3} \Delta \]

\[ F-H \]
\[ \Delta \]
\[ R^* \]
\[ \frac{\Delta}{R^*} \]
\[ F^* \]
\[ \frac{F^*}{L} \]

\[ \{ f' \}_{F-H} = \begin{bmatrix} 0 & F^* & R^* & 0 & -F^* & 0 \end{bmatrix}^T \]

\[ R^* = \frac{3EI}{L^2} \Delta, \quad F^* = \frac{3EI}{L^3} \Delta \]
Space Truss

General Argument:
(See NOTES below)

Plane of k force

(local)

$P = \{P_x, P_y, P_z\}^T$
$P_a$: axial component of $P$
$P_n$: normal component of $P$

$P_a = \{P_{ax}, P_{ay}, P_{az}\}^T = \{P_{alx}, P_{amlx}, P_{anx}\}^T$
$P_a = |P_a|$

$P_n = P - P_a = \{(P_x - P_{ax})_x, (P_y - P_{ay})_y, (P_z - P_{az})_z\}^T = \{P_{nx}, P_{ny}, P_{nz}\}^T$

If prismatic:
$F_1 = P_a \frac{b}{L}$, $F_2 = P_a \frac{a}{L}$

$Q_1 = \{P_{ax} \frac{b}{L}, P_{ay} \frac{b}{L}, P_{az} \frac{b}{L}\}^T$

$Q_2 = \{P_{ax} \frac{a}{L}, P_{ay} \frac{a}{L}, P_{az} \frac{a}{L}\}^T$

$\{f\}' = \{F_1, 0, F_2, 0\}^T$

NOTE: EFFECT OF $P_n$ SHOULD BE NOT INTRODUCED DIRECTLY AT THE JOINTS
BECAUSE NO LOCAL AXES ARE DEFINED NORMAL TO THE MEMBER.
Direction of components of joint movements

A) Joint Displacements for Different Models

1) Plane Truss

2) Plane Frame

3) grid

4) Space Truss

5) Space Frame

In general, at each joint there are 6 DOFs (3 translations + 3 rotations). In different models, some of these DOFs are absent either because they are prevented (e.g. out-of-plane displacements in plane frames and plane trusses, or in-plane displacements in grids) or because occurrence of such DOFs produces no internal forces, and therefore such displacements do not appear in the equilibrium equations and cannot be determined (e.g. rotation about an axis normal to the plane of a truss).
B. Local Versus Global Directions

In the analysis of any structure by the stiffness method, a fixed system of axes is taken for the whole structure (GLOBAL SYSTEM), while another system is assumed for each of the members of the structure (LOCAL SYSTEMS).

Example:

\[ x_L, y_L, z_L \] are the local system of axes for member AB.
\[ x_G, y_G, z_G \] are the global system of axes for the structure.

→ Representing displacements in "global directions" is useful when studying joints equilibrium.
→ Representing displacements in "local directions" is useful when studying the individual members to determine member end-forces due to member loads considering restrained joints (recall case (a)).