The 11th US National Congress on Computational Mechanics

Reliability Based Topology Optimization under Stochastic Excitation

Junho Chun

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Advisors : Junho Song & Glaucio H. Paulino

Department of Civil and Environmental Engineering
University of Illinois, Urbana-Champaign

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- Discrete representation of a random process
- Reliability based topology optimization
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- Future research

Introduction

- Optimization in structural engineering
- Motivation
- Stochastic Process

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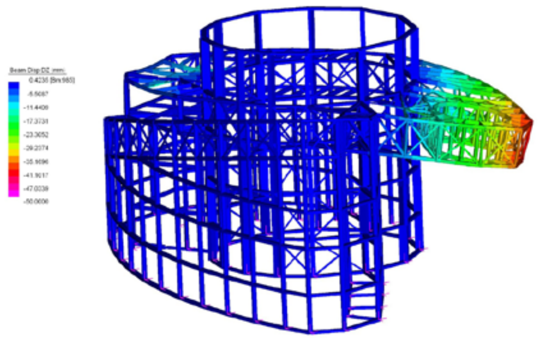
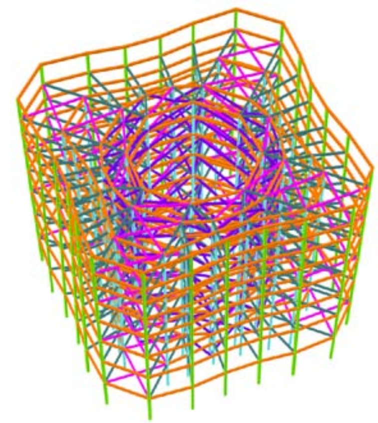
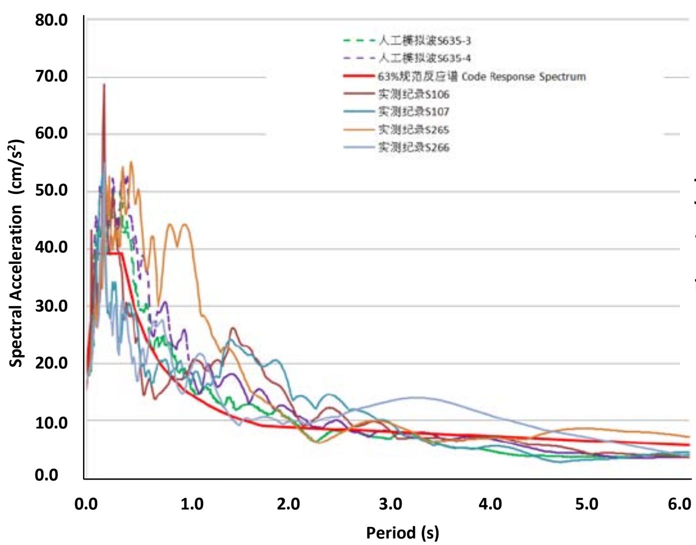
Optimization in structural engineering

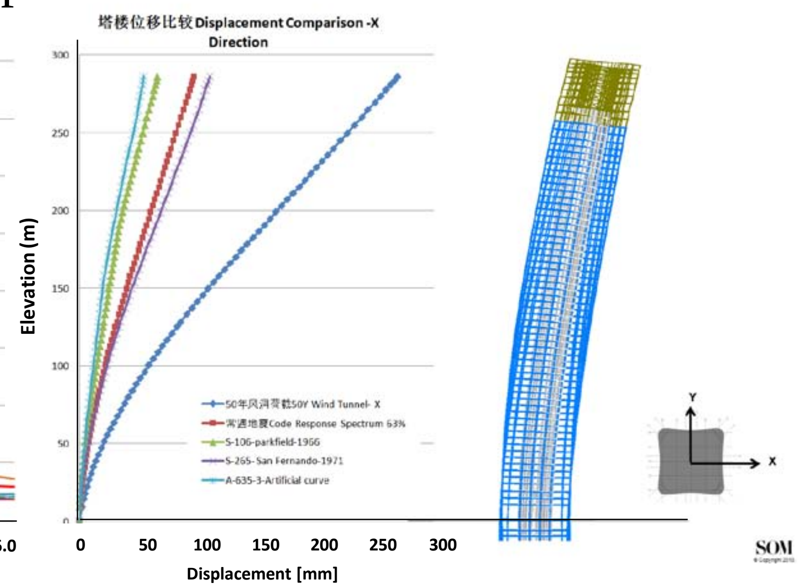
- Structural elements optimization





- Structure performance optimization



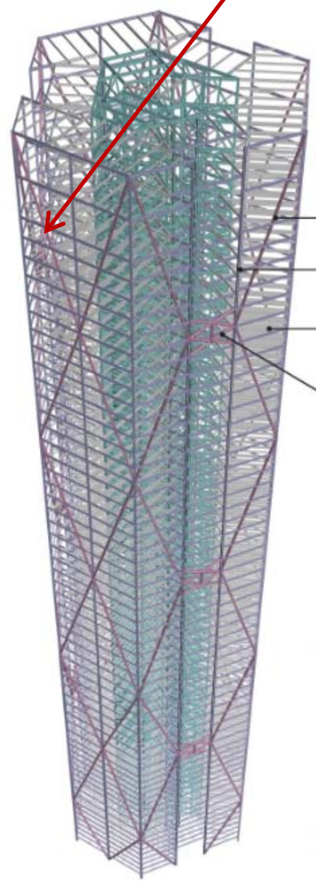


Motivation

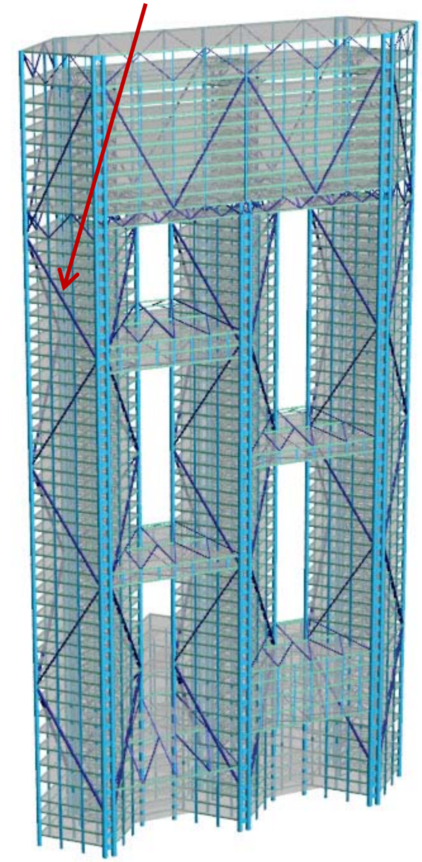
- Application of Topology optimization under stochastic process to building system design

JOHN HANCOCK
Courtesy of Skidmore, Owing and Merrill, LLP

MEGA-BRACE DIAGONALS



BBVA

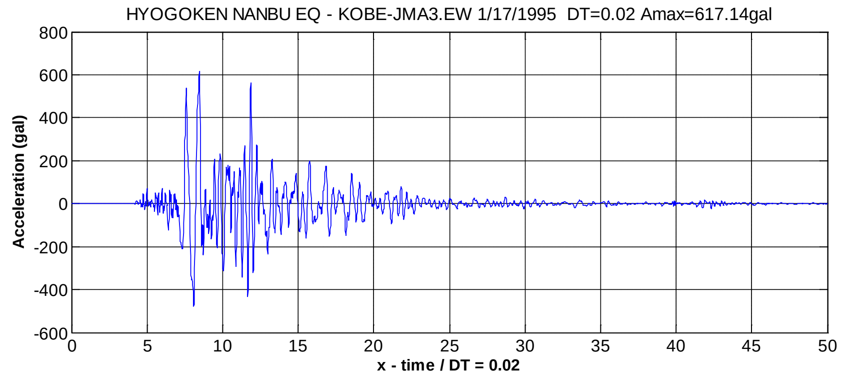


SSIGER

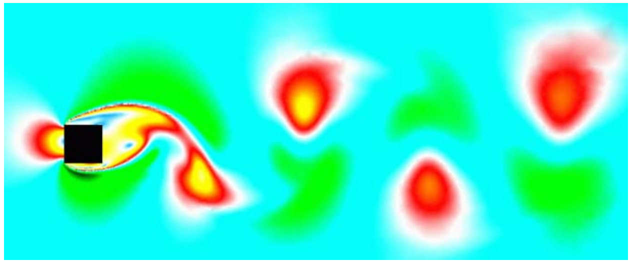
Stochastic process

- Random process
- Non-deterministic excitations
- Many possibilities the process might to go to

- Earthquake excitations



- Wind loads



Discrete Representation of a Random Process

- Discrete representation
- Response
- Probability of failure

Discrete representation

Stochastic excitation can be discretized and represented in terms of a finite number of standard normal random variables

$$f(t) = \mu(t) + \sum_{i=1}^{n} u_i s_i(t) = \mu(t) + \mathbf{s}(t)^{\mathrm{T}}\mathbf{u}$$

- ► Mean function , $\mu(t)$
- ► Standard normal random variables, $\mathbf{u} = [\ u_1 \ldots u_n\ ]^{\mathrm{T}}$
- ► Deterministic basis functions, $\mathbf{s}(t) = [\ s_1(t) \ldots s_n(t)\ ]^{\mathrm{T}}$

  - Dependent on the covariance structure of the process

    - Karhunen – Loeve expansion method

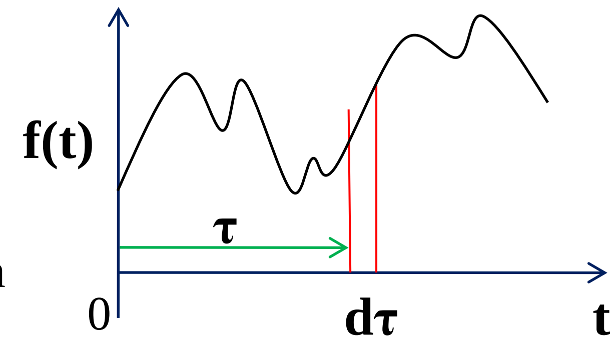
      $$s_i(t) = \sqrt{\lambda_i}\,\psi_i(t)$$

    - etc.

Linear system + Gaussian / Non-Gaussian process

- Duhamel's Integral

$$x(t) = \int_0^t f(\tau)h(t-\tau)\,\mathrm{d}\tau$$



- ***h(t)*** : the unit-impulse response function of the system

- Stochastic excitation

$$f(t) = \mu(t) + \sum_{i=1}^{n} u_i s_i(t) = \mu(t) + \mathbf{s}(t)^{\mathrm{T}}\mathbf{u}$$

- Response

$$x(t) = \int_0^t \sum_{i=1}^{n} u_i s_i(\tau) h(t-\tau)\,\mathrm{d}\tau = \sum_{i=1}^{n} u_i a_i(t) = \mathbf{a}(t)^{\mathrm{T}}\mathbf{u},$$

$$\mathbf{a}(t) = [\, a_1(t) \ldots a_n(t) \,]^{\mathrm{T}} \qquad a_i(t) = \int_0^t s_i(\tau)h(t-\tau)\,\mathrm{d}\tau$$

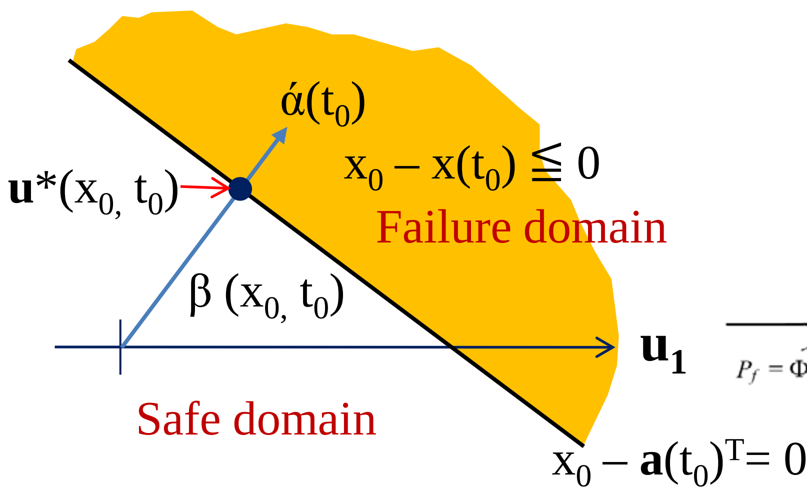
Limit state function

Consider the set of realizations of $\boldsymbol{f(t)}$ that give rise to the event $\{x(t_0) \geq x_0\}$ at time $t = t_0$, where $\mathbf{x_0}$ is a selected threshold

- Realizations of **u** that satisfy the condition

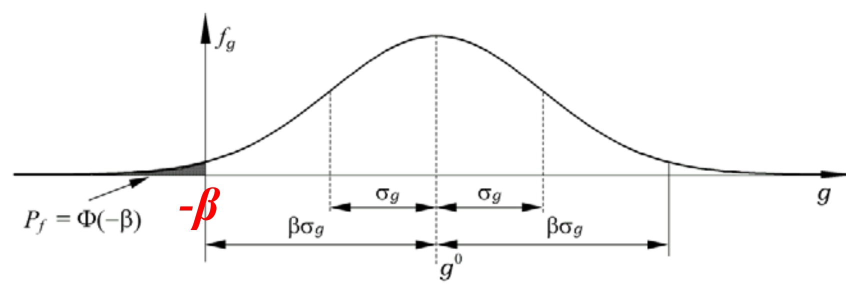
  $$G : x_0 - \mathbf{a}(t_0)^T \mathbf{u} \leqq 0, \text{ failure event}$$

  - In the space of **u**, these lie in a half space bounded by the hyper-plane, $x_0 - \mathbf{a}(t_0)^T \mathbf{u} = 0$



Geometric representation of response at time t_{0v}

- Reliability index
  - $\beta(x_0,t_0) = x_0 / \|\mathbf{a}(t_0)\|$



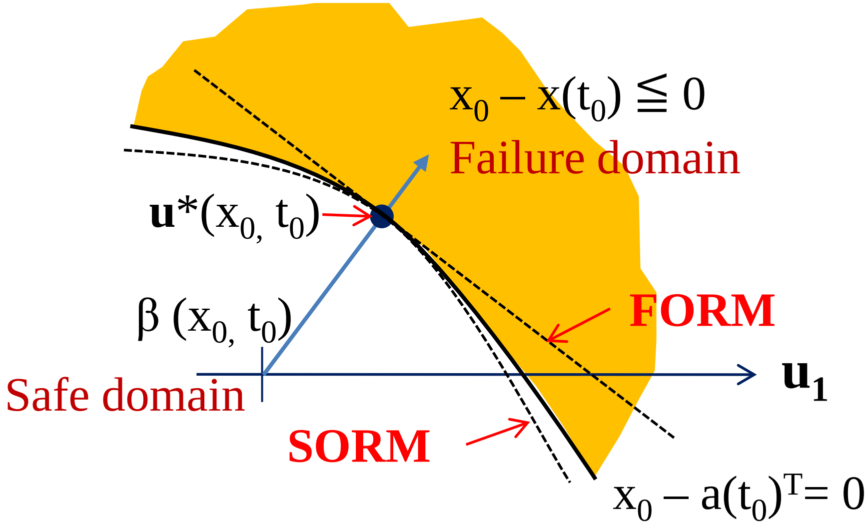
- Failure probability
  - $Pf\,[\,x(t_0) > x_0\,] = \Phi\,[\,-\beta(x_0,t_0)]$

FORM / SORM approximation

To evaluate limit state surface of Non-Gaussian response or nonlinear system

- FORM (First Order Reliability Method)
  Linearizing limit state function in the standard normal space at an optimal point

- SORM (Second Order Reliability Method)
  Second order approximation of the limit state function



FORM and SORM approximations for non-Gaussian response

Reliability Based Topology Optimization

- Concept
- CRBTO / SRBTO
- Formulation of the RBTO problem
- Formulation of the optimization problem (on going research)

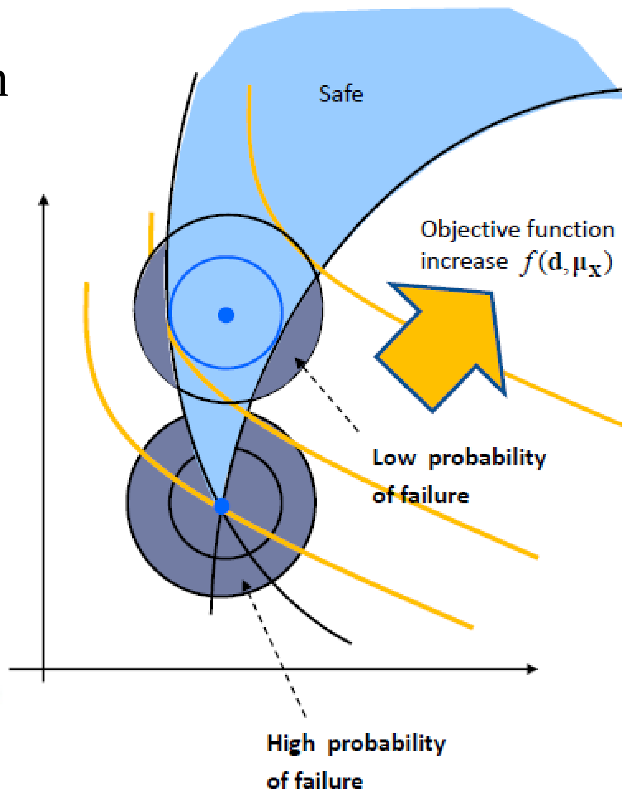
Aim to obtain that maximizes the performance under the design constrains on the failure probabilities

- Deterministic topology optimization

$$\min_{\mathrm{d}} \quad f(\mathrm{d})$$
$$s.t. \quad \mathrm{g_i}(\mathrm{d}, \mathrm{X}) > 0$$
$$\mathrm{d^L} \leq \mathrm{d} \leq \mathrm{d^U}$$

- Reliability based topology optimization (RBTO)

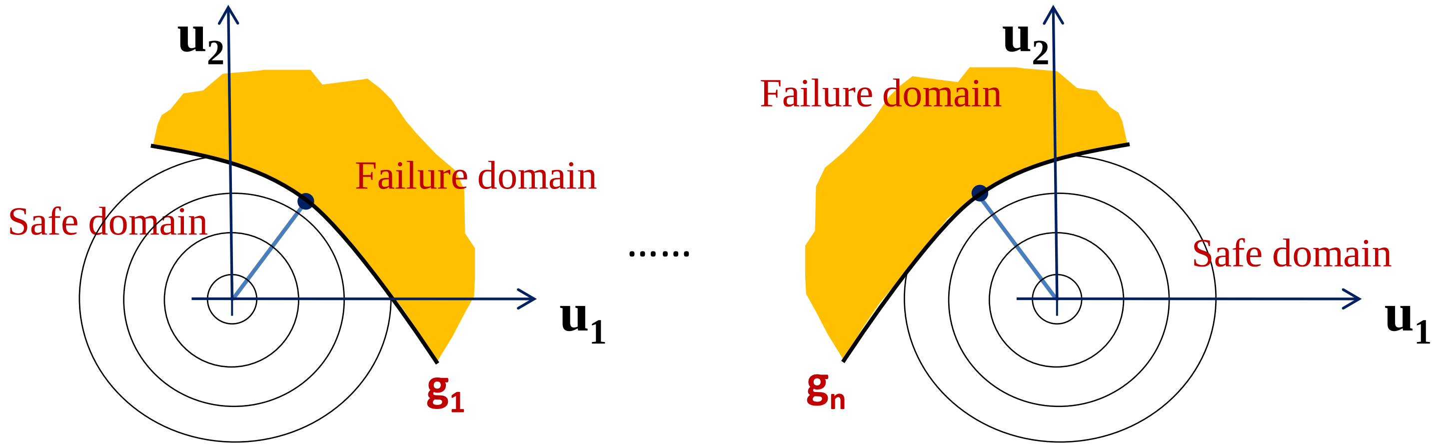
$$\min_{\mathrm{d}} \quad f(\mathrm{d})$$
$$s.t. \quad P\left[g_i(\mathbf{d}, \mathbf{X}) \leq 0\right] \leq P_i^t, \quad i=1,\ldots,n$$



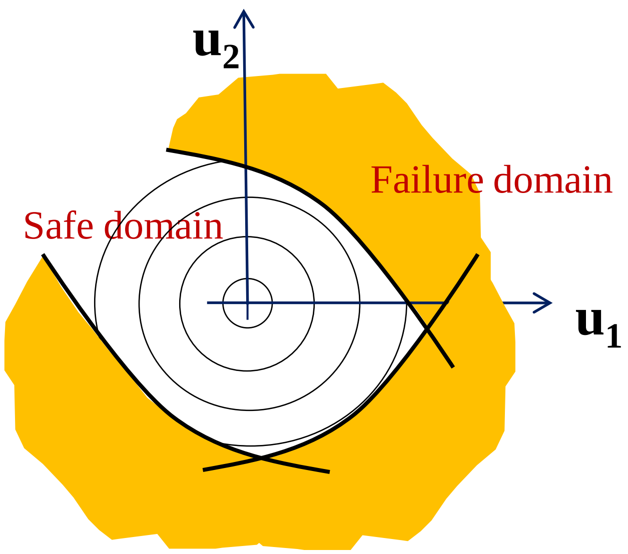
Nguyen, T.H., System reliability-based design and multi-resolution topology optimization, PhD Thesis (2010)

Reliability Based Topology Optimization

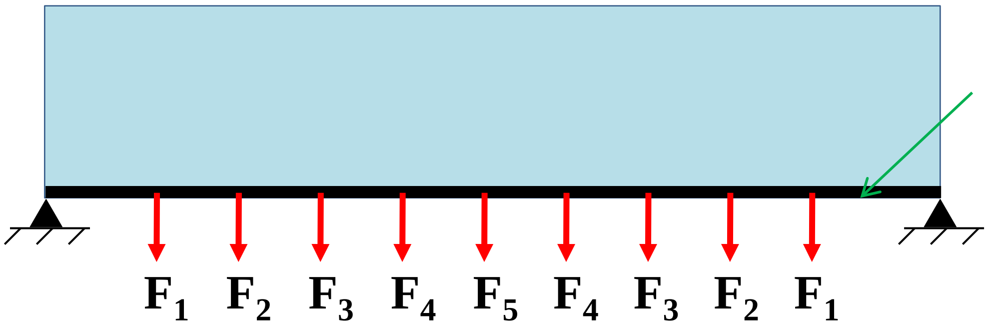
- Component RBTO



- System RBTO



Formulation of the RBTO problem



$$\min_{\rho} : V(\rho)$$

$$g_i(\boldsymbol{\rho},\mathbf{F}) = d_i^0 - d_i(\rho,F), \quad i = 1,..,5$$

$$d_i^0 = \{1.25, 1.50, 1.75, 2.00, 2.20\}, \ i=1,\ldots,5$$

Each of random variables : $\boldsymbol{\mu}_\mathbf{F} = 10^5$

c.o.v $(\boldsymbol{\sigma}_\mathbf{F}/\boldsymbol{\mu}_\mathbf{F}) = 1/6$

Nguyen, T.H., J. Song, and G.H. Paulino (2010). Single-loop system reliability-based design optimization using matrix-based system reliability method: theory and applications. J. of Mechanical Design, ASME, Vol. 132, 011005-1~11.

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Results

A

B

C

Courtesy of Nguyen (2011)

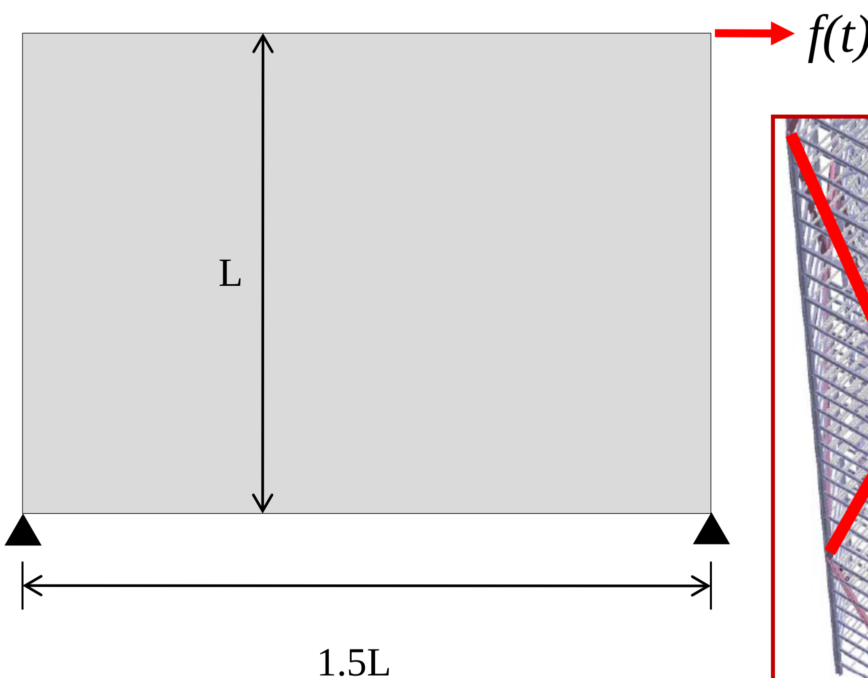
(A)	(B)	(C)
DTO (μF $=10^5$)	FORM-based CRBTO ((μF $=10^5$)	FORM-based SRBTO ((μF $=10^5$)
volfrac = 39.07%	$P_i^t = 0.02275$, $P_{sys}^t = 0.06657$ volfrac = 48.64%	$P_{sys}^t = 0.06657$, volfrac = 47.7%

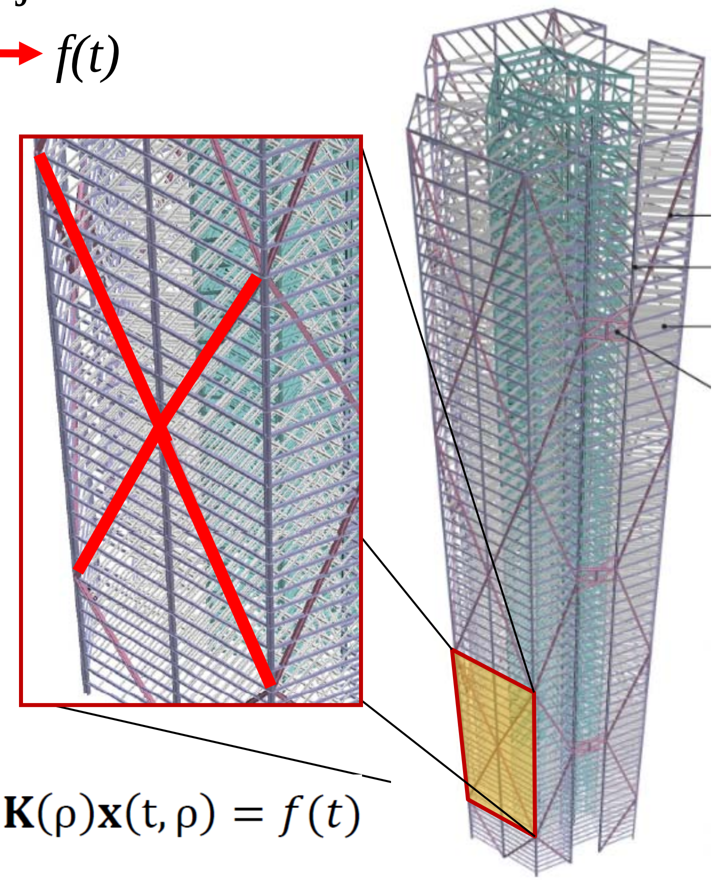
Nguyen, T.H., J. Song, and G.H. Paulino (2010). Single-loop system reliability-based design optimization using matrix-based system reliability method: theory and applications. J. of Mechanical Design, ASME, Vol. 132, 011005-1~11.

Reliability Based Topology Optimization

Formulation of the optimization problem (on going)

Design of a rectangular domain subjected to stochastic excitations





$$\min_{\rho} : V(\rho)$$

$$\text{s.t.} : \mathbf{M}(\rho)\ddot{\mathbf{x}}(t,\rho) + \mathbf{C}(\rho)\dot{\mathbf{x}}(t,\rho) + \mathbf{K}(\rho)\mathbf{x}(t,\rho) = f(t)$$

$$Pf\,[\,G: x_0 - \mathbf{a}(t_0)^T \mathbf{u} \leqq 0\,] \leqq Pf^{\,target}$$

$$\Phi\,[\,-\beta(x_0,t_0)] \leqq \Phi\,[\,-\beta^{\,target}(x_0,t_0)]$$

x_0 : threshold value

t_0 : time

Sensitivity

- Constraint function
  - The sensitivities with respect to the design variables, **ρ** obtained by applying the chain rule :

$$Pf\,[\,G: x_0 - \mathbf{a}(t_0)^T\mathbf{u} \leqq 0\,] \leqq Pf^{\,target}$$

$$\frac{\partial pf}{\partial \boldsymbol{\rho}}(G: x_0 - \mathbf{a}(t_0)^T\mathbf{u} \leq 0) = \varphi(-\beta)\frac{\partial \beta}{\partial \rho}$$

, where $\beta(x_0,t_0) = x_0 / \|\mathbf{a}(t_0)\|$

Summary

- Discrete Representation of stochastic process, response was presented
- Probability of failure is obtained from discrete representation easily
- Reliability based topology optimization for static loads problem was reviewed.
- Reliability based topology optimization for stochastic loads case is on going research

Future Research

- Implementation topology optimization with stochastic process
- Nonlinear system
- Earthquake ground motion modeling

  Sanaz Rezaeian, Armen Der Kiureghian (2010) Simulation of synthetic ground motions for specified earthquake and site characteristics
- The first passage probability

  Song, J., and A. Der Kiureghian (2006). Joint first-passage probability and reliability of systems under stochastic excitation. Journal of Engineering Mechanics. ASCE, 132(1), 65-77
- Application to multiple story building system optimization

Thanks