A Unified Library of Nonlinear Solution Schemes

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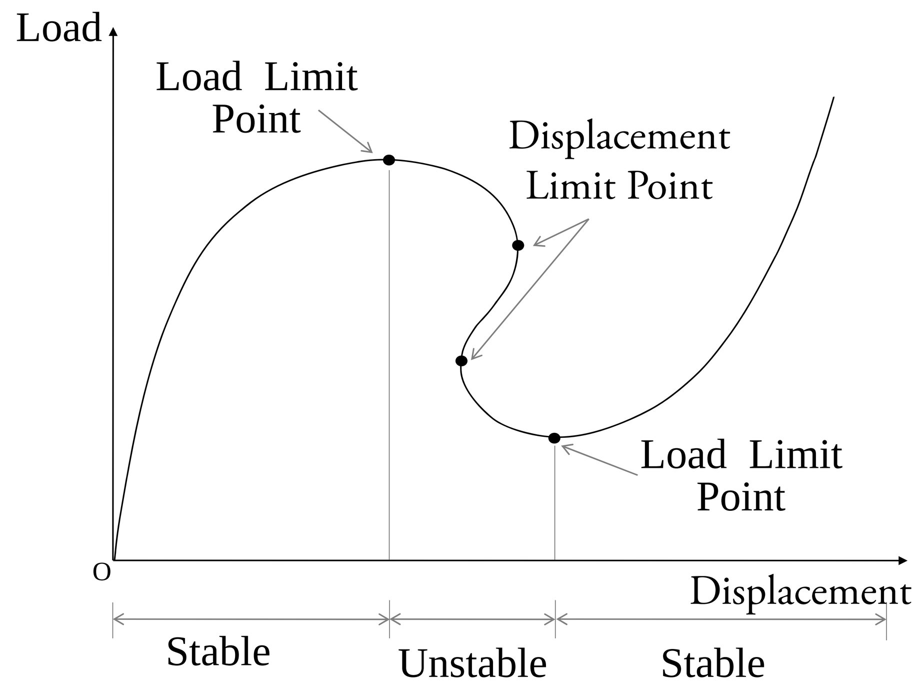
Motivation

- Nonlinear problems are prevalent in structural and continuum mechanics
- No single algorithm is appropriate for solving any and all nonlinear problems
- A library of nonlinear solution schemes, defined by unique constraint equations, is unified into a single space

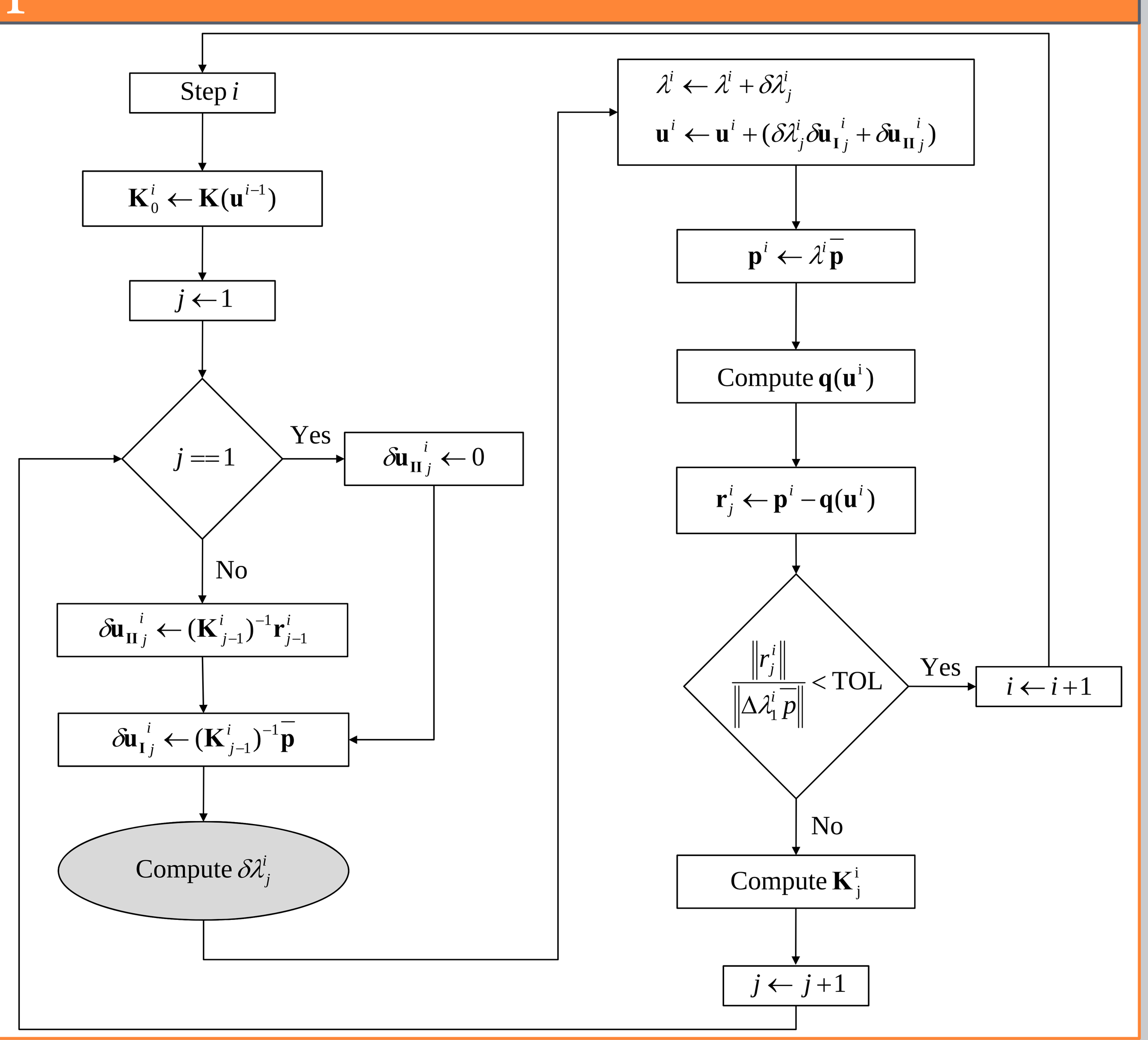


N+1 Dimensional Space Formulation: NLS++

$$\mathbf{u}_j = \mathbf{u}^{prev} + \Delta\mathbf{u}_{j-1} + \delta\mathbf{u}_j$$
$$\mathbf{p}_j = \mathbf{p}^{prev} + \Delta\mathbf{p}_{j-1} + \delta\mathbf{p}_j$$

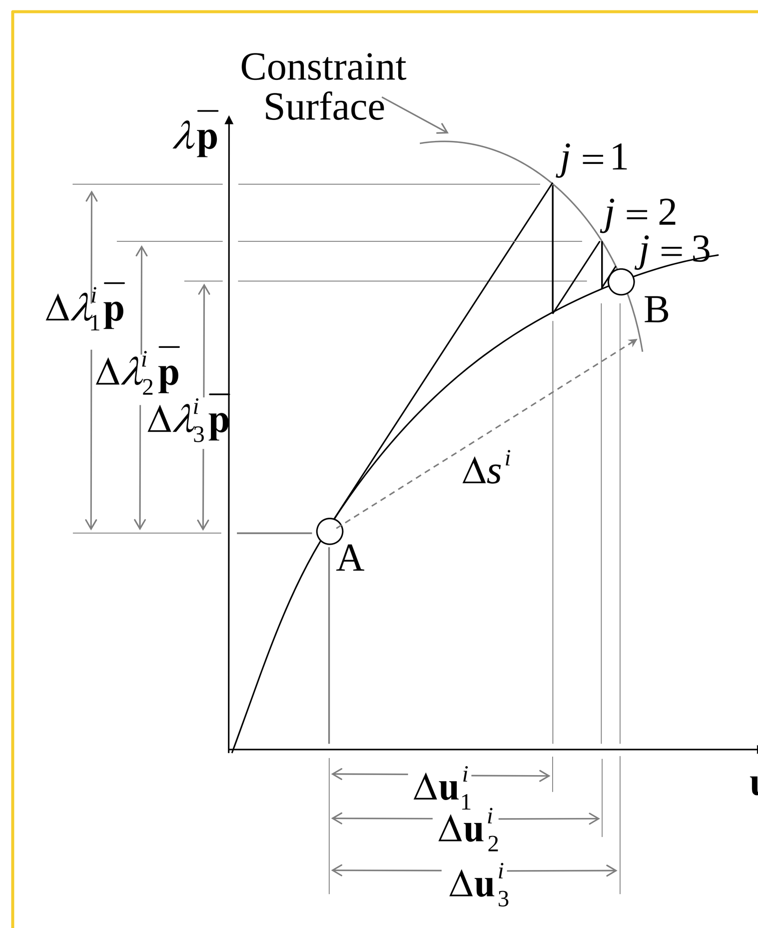
$$\mathbf{p}_j = \mathbf{p} + \Delta\mathbf{p}_{j-1} + \delta\lambda_j\bar{\mathbf{p}}$$
$$\mathbf{a}_j \cdot \delta\mathbf{u}_j + b_j\delta\lambda_j = c_j$$
$$\begin{bmatrix} \mathbf{K}_{j-1} & -\bar{\mathbf{p}} \\ (\mathbf{a}_j)^T & b_j \end{bmatrix} \begin{Bmatrix} \delta\mathbf{u}_j \\ \delta\lambda_j \end{Bmatrix} = \begin{Bmatrix} \mathbf{r}_{j-1} \\ c_j \end{Bmatrix}$$

$$\delta\mathbf{u} = \delta\mathbf{u}_I + \delta\lambda\delta\mathbf{u}_{II}$$
$$\mathbf{K}_{j-1}\delta\mathbf{u}_{j_I} = \bar{\mathbf{p}}$$
$$\mathbf{K}_{j-1}\delta\mathbf{u}_{j_{II}} = \mathbf{r}_{j-1}$$
$$\delta\lambda_j = \frac{c_j - \mathbf{a}_j \cdot \delta\mathbf{u}_{j_{II}}}{\mathbf{a}_j \cdot \delta\mathbf{u}_{j_{II}} + b_j}$$

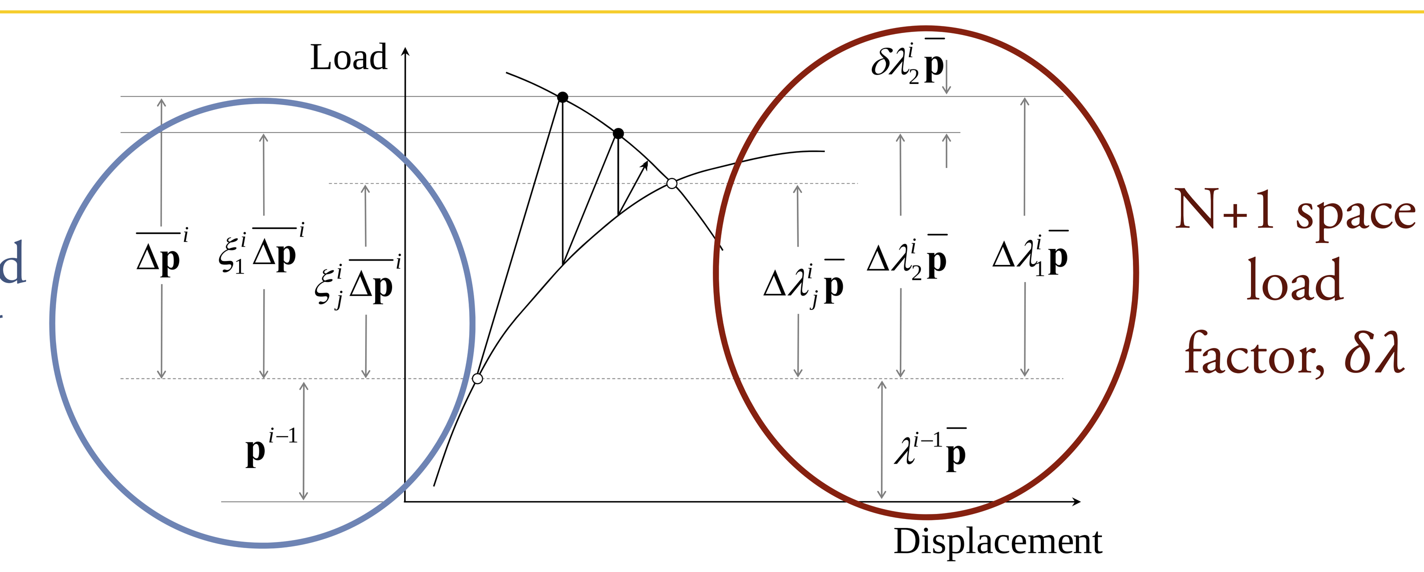


Solution Schemes

- $\delta\lambda$ is computed uniquely for each nonlinear solution scheme:
  - Load control method (LCM)
  - Displacement control method (DCM)
  - Arc-length control method (ALCM)
  - Work control method (WCM)
  - Generalized displacement control method (GDCM)
  - Orthogonal residual procedure (ORP)



Arc-length control method

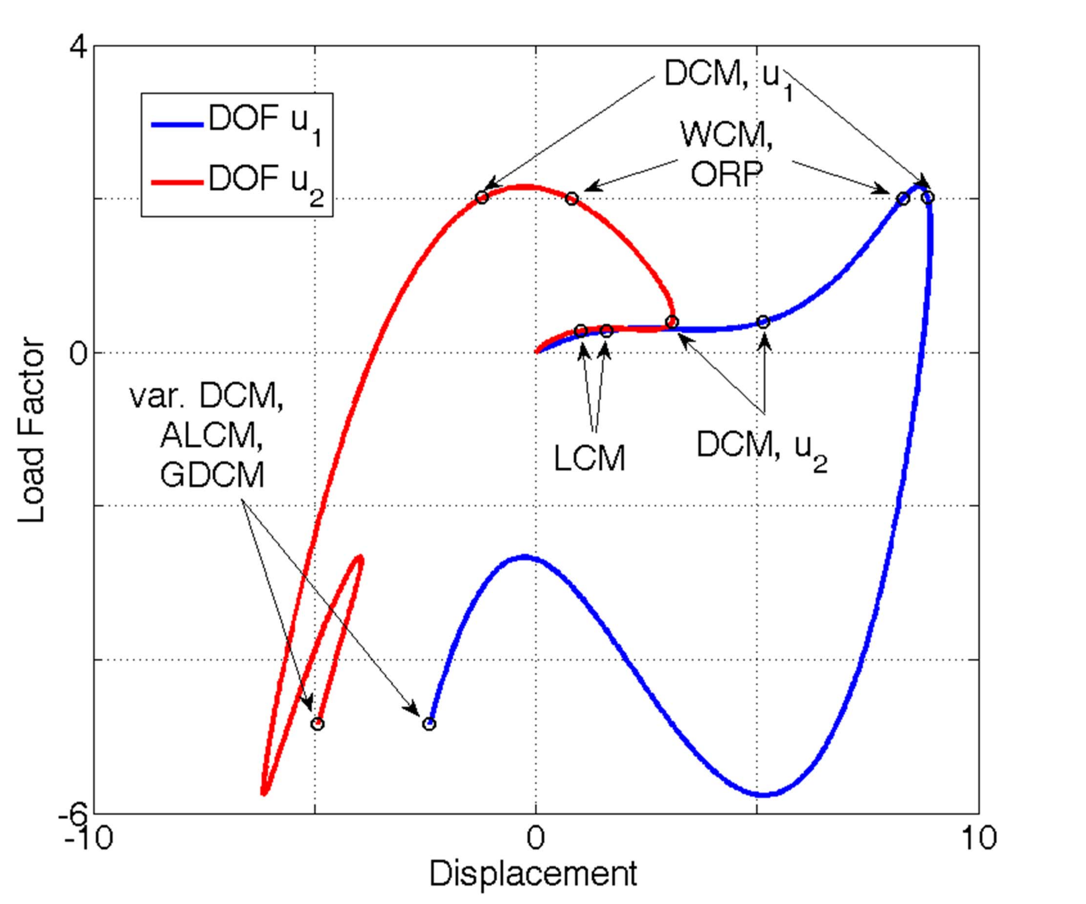


$$\mathbf{p}^{i-1} + \xi_j^i\overline{\Delta\mathbf{p}} = \lambda^{i-1}\bar{\mathbf{p}} + \Delta\lambda_{j-1}^i\bar{\mathbf{p}} + \delta\lambda_j^i\bar{\mathbf{p}}$$
$$\delta\lambda_j^i = \frac{\left[\mathbf{q}\left(\mathbf{u}^{i-1} + \Delta\mathbf{u}_j^i\right)\right] \cdot \Delta\mathbf{u}_j^i}{\bar{\mathbf{p}} \cdot \Delta\mathbf{u}_j^i} - \left(\lambda^{i-1} + \Delta\lambda_{j-1}^i\right)$$

Numerical Results

Two Dimensional Function

$$q(u) = \begin{pmatrix} 10u_1 + 0.4u_2^3 - 5u_2^2 \\ 0.4u_1^3 - 3u_1^2 + 10u_2 \end{pmatrix}$$



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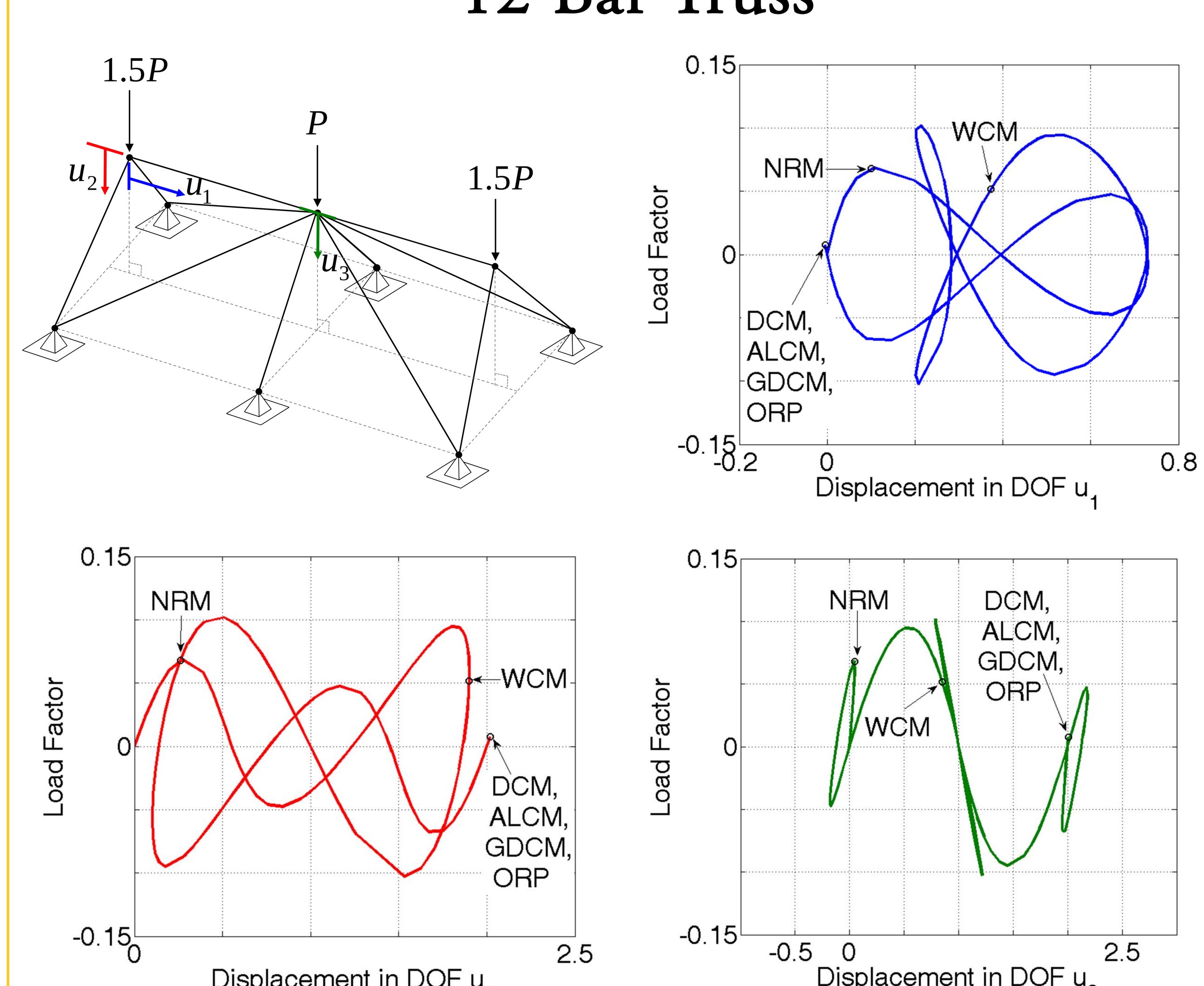
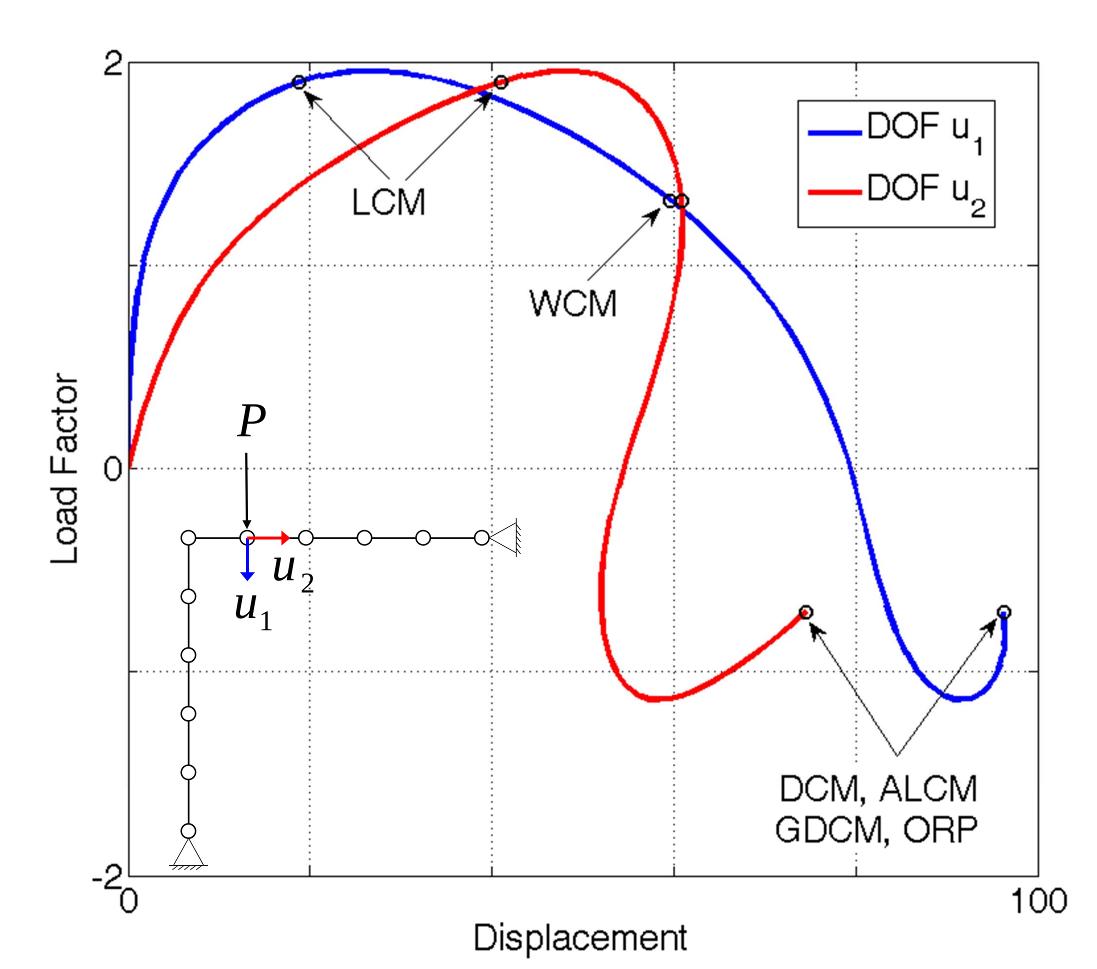


Lee Frame



Conclusions and Extensions

- NLS++ is an effective computational framework for solving problems with varying degrees of nonlinearity
- Robustness of algorithms is evaluated by means of the unified schemes
- Potential to incorporate NLS++ into an object-oriented finite element engine

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