Mechanical and Thermal Buckling of Functionally Graded Plates

Roman Arciniega and J. N. Reddy

Department of Mechanical Engineering
Texas A&M University
College Station, TX 77843-3123, USA

US-South American Workshop:
Mechanics and Advanced Materials
– Research and Education
August 2-6, 2004
CONTENTS OF THE LECTURE

• Background
• Theoretical Formulation
• FE Model
• Numerical Results
• Closing comments
Functionally graded materials are inhomogeneous materials in which the material properties are varied continuously from point to point to eliminate interface problems and thus the stress distributions are smooth and uniform.

For example, a plate structure used as a thermal barrier may be graded through the plate thickness from ceramic on the face of the plate that is exposed to high temperature to metal on the other face.
\[ P(z) = (P_c - P_m) f(z) + P_m \]
\[ f(z) = \left[ \frac{(2z + h)}{2h} \right]^n \]
This is achieved by varying the volume fraction of the constituents i.e., ceramic and metal in a predetermined manner. The ceramic constituent of the material provides the high temperature resistance due to its low thermal conductivity. The ductile metal constituent, on the other hand, prevents fracture caused by stresses due to high temperature gradient in a very short period of time.
<table>
<thead>
<tr>
<th>High temperature side</th>
<th>Ceramic</th>
<th>Heat resistant; good anti-oxidant property; low thermal conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low temperature side</td>
<td>Metal</td>
<td>Mechanical strength; high thermal conductivity; high fracture toughness</td>
</tr>
<tr>
<td>In between</td>
<td>Ceramic &amp; metal</td>
<td>Effective thermal stress relaxation throughout</td>
</tr>
</tbody>
</table>

**Diagram:**
- $P_c$ (ceramic)
- $P_m$ (metal)
Aluminum and Zirconia
Material Properties

**Aluminum**

\[ E_1 = 70 \text{ GPa}, \quad \nu = 0.3, \quad \rho = 2707 \text{ kg/m}^3, \]
\[ k = 204 \text{ W/(m.K)}, \quad \alpha = 23 \times 10^{-6} / ^\circ\text{C} \]

**Zirconia**

\[ E_1 = 151 \text{ GPa}, \quad \nu = 0.3, \quad \rho = 3000 \text{ kg/m}^3, \]
\[ k = 2.09 \text{ W/(m.K)}, \quad \alpha = 23 \times 10^{-5} / ^\circ\text{C} \]
The temperature distribution over the thickness is determined by solving the energy equation

\[- \frac{d}{dz} \left( k(z) \frac{dT}{dz} \right) = 0 \]

Non-dimensional parameters

Center deflection:

\[ \overline{w} = \frac{w}{h} \]

Load parameter:

\[ P = \frac{q_o a^4}{E_m h^4} \]
Temperature variation through the aluminum-zirconia FGM plate thickness
SUMMARY

- The mechanical and thermal buckling of functionally graded ceramic-metal plates is investigated.
- The third-order shear deformation theory for plates is used.
- A displacement finite element model of the third-order theory is developed using $c^0$-continuity with a family of high-order Lagrange interpolation functions to avoid shear locking.
- The stability equations are derived using the Trefftz criterion.
- Numerical results are compared with those of the FSDT.
Most of research studies in FGMs had more focused on thermal stress analysis and fracture mechanics (Paulino and his colleagues).

Limited work has been done to study the buckling and vibration response of FGM structures.

Javaheri and Eslami derived the stability equations of FGM plates under thermal loads, based on the CLPT.

Shen carried out a postbuckling analysis for FGM panels and plates subjected to axial compression in thermal environments.

Na and Kim presented a 3D finite element solution for thermal buckling of functionally graded plates.
Displacement Field of TSDT

\[ u_1(x, y, z) = u + z \varphi_1 + k (w, x + \varphi_1) \]
\[ u_2(x, y, z) = v + z \varphi_2 + k (w, y + \varphi_2) \]
\[ u_3(x, y, z) = w \]
To relax the continuity in the finite element formulation, we introduce the following auxiliary variables

\[
\psi_1 = w_x + \varphi_1 \\
\psi_2 = w_y + \varphi_2
\]

The strains are

\[
\varepsilon_i = \varepsilon_i^\circ + k_i^1 z + k_i^3 z^3 \\
\varepsilon_m = \varepsilon_m^\circ + k_m^2 z^2
\]

\[i = 1, 2, 6\]  
\[m = 4, 5\]
where

\[ \varepsilon_1^0 = u_x + \frac{1}{2} w_x^2, \quad \varepsilon_2^0 = v_y + \frac{1}{2} w_y^2, \quad \varepsilon_5^0 = w_{,x} + \varphi_1 \]

\[ \varepsilon_4^0 = w_{,y} + \varphi_2, \quad \varepsilon_6^0 = u_{,y} + v_{,x} + w_{,x} w_{,y}, \]

and

\[ k_1^1 = \varphi_{1,x}, \quad k_2^1 = \varphi_{2,y}, \quad k_6^1 = \varphi_{1,y} + \varphi_{2,x}, \]

\[ k_1^3 = k \psi_{1,x}, \quad k_2^3 = k \psi_{2,y}, \quad k_6^3 = k (\psi_{1,y} + \psi_{2,x}), \]

\[ k_4^2 = 3k \psi_{2,y}, \quad k_5^2 = 3k \psi_{1,x} \]
EQUILIBRIUM EQUATIONS

\[ \delta u : \quad \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0 \]
\[ \delta v : \quad \frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = 0 \]
\[ \delta w : \quad \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + q + \bar{N} = 0 \]

\[ \delta \phi_1 : \quad \frac{\partial M_1}{\partial x} + \frac{\partial M_6}{\partial y} - Q_1 = 0 \]
\[ \delta \phi_2 : \quad \frac{\partial M_6}{\partial x} + \frac{\partial M_2}{\partial y} - Q_2 = 0 \]
\[ \delta \psi_1 : \quad \frac{\partial P_1}{\partial x} + \frac{\partial P_6}{\partial y} - 3 R_1 = 0 \]
\[ \delta \psi_2 : \quad \frac{\partial P_6}{\partial x} + \frac{\partial P_2}{\partial y} - 3 R_2 = 0 \]

\[ (N_i, M_i, P_i) = \int_{-h/2}^{h/2} \sigma_i(1, z, z^3) \, dz \quad (i = 1, 2, 6), \]

\[ (Q_1, R_1) = \int_{-h/2}^{h/2} \sigma_5(1, z^2) \, dz, \quad (Q_2, R_2) = \int_{-h/2}^{h/2} \sigma_4(1, z^2) \, dz \]
MECHANICAL CHARACTERIZATION OF FUNCTIONALLY GRADED PLATES

\[ E(z) = E_c f_c + E_m f_m = E_{cm} f_c + E_m \]

\[ \alpha(z) = \alpha_c f_c + \alpha_m f_m = \alpha_{cm} f_c + \alpha_m \]

Volume Fractions:

\[ f_c = \left( \frac{z + \frac{1}{2}}{h} \right)^n \quad f_m = 1 - f_c \]

Constant Poisson’s coefficient \( \nu(z) = \nu \)
CONSTITUTIVE EQUATIONS:

\[
\begin{align*}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix} &= \begin{bmatrix}
Q_{11}(z) & Q_{12}(z) & 0 \\
Q_{12}(z) & Q_{22}(z) & 0 \\
0 & 0 & Q_{66}(z)
\end{bmatrix} \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_6
\end{bmatrix} \\
\begin{bmatrix}
\sigma_4 \\
\sigma_5
\end{bmatrix} &= \begin{bmatrix}
Q_{44}(z) & 0 \\
0 & Q_{55}(z)
\end{bmatrix} \begin{bmatrix}
\varepsilon_4 \\
\varepsilon_5
\end{bmatrix}
\end{align*}
\]

WHERE:

\[
Q_{11}(z) = Q_{22}(z) = \frac{E(z)}{1-\nu^2}, \quad Q_{12}(z) = \frac{\nu E(z)}{1-\nu^2}, \\
Q_{66}(z) = Q_{44}(z) = Q_{55}(z) = \frac{E(z)}{2(1+\nu)}
\]
STRESS RESULTANTS:

\[ N_i = A_{ij} \varepsilon_j^0 + B_{ij} k_j^1 + E_{ij} k_j^2 \]

\[ M_i = B_{ij} \varepsilon_j^0 + D_{ij} k_j^1 + F_{ij} k_j^3 \]

\[ P_i = E_{ij} \varepsilon_j^0 + F_{ij} k_j^1 + H_{ij} k_j^3 \]

\[ N_i = A_{ij} \varepsilon_j^0 + B_{ij} k_j^1 + E_{ij} k_j^2 \]

\[ M_i = B_{ij} \varepsilon_j^0 + D_{ij} k_j^1 + F_{ij} k_j^3 \]

\[ P_i = E_{ij} \varepsilon_j^0 + F_{ij} k_j^1 + H_{ij} k_j^3 \]

\[ Q_1 = \overline{A}_{ij} \varepsilon_j^0 + \overline{D}_{ij} k_j^2 \]

\[ Q_2 = \overline{A}_{ij} \varepsilon_j^0 + \overline{D}_{ij} k_j^2 \]

\[ R_1 = \overline{D}_{ij} \varepsilon_j^0 + \overline{F}_{ij} k_j^2 \]

\[ R_2 = \overline{D}_{ij} \varepsilon_j^0 + \overline{F}_{ij} k_j^2 \]

THE MATERIAL STIFFNESS COEFFICIENTS FOR FGMs:

\[(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} (Q_{ij}^m f_c + Q_{ij}^m)(1, z, z^2, z^3, z^4, z^6) dz\]

\[(\overline{A}_{ij}, \overline{D}_{ij}, \overline{F}_{ij}, \overline{}) = \int_{-h/2}^{h/2} (Q_{ij}^m f_c + Q_{ij}^m)(1, z, z^4) dz\]
RECALL: \[ Q_{ij}^{cm} = Q_{ij}^c - Q_{ij}^m \]

THE THERMAL STRESSES ARE:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix}^T = -\begin{bmatrix}
Q_{11}(z) & Q_{12}(z) & 0 \\
Q_{12}(z) & Q_{22}(z) & 0 \\
0 & 0 & Q_{66}(z)
\end{bmatrix}\begin{bmatrix}
\alpha(z) \\
\alpha(z) \\
0
\end{bmatrix}T(x, y, z)
\]

THERMAL STRESS RESULTANTS:

\[
(N^T, M^T, P^T) = -\frac{1}{(1-\nu)} \int_{-h/2}^{h/2} (E_{cm}f_c + E_m)(\alpha_{cm}f_c + \alpha_m)T(x, y, z)(1, z, z^2)dz
\]
The potential energy increment may be written in the form

$$\delta \Pi = \Pi(U^F + U^I) - \Pi(U^F) = \sum_{n=1}^{4} \frac{1}{n!} \delta^n \Pi$$

**U^F**: Fundamental prebuckling solution

**U^I**: Incremental solution

*Necessary condition*  \(-\delta \Pi = 0\)
The critical load is defined as the smallest load for which the second variation is no longer positive definite. The limit of positive-definiteness for a continuous system can be expressed as

\[ \delta \left[ \delta^2 \Pi \right] = 0 \]

(THE TREFFTZ CRITERION)

The fundamental solution (prebuckling state) is considered as a pure membrane state where bending and rotations are neglected. Then

\[
U \rightarrow \begin{cases} 
  u = u^0 + u^1, & v = v^0 + v^1, & w = w^1, \\
  \varphi_1 = \varphi_1^1, & \varphi_2 = \varphi_2^1, \\
  \psi_1 = \psi_1^1, & \psi_2 = \psi_2^1 
\end{cases}
\]
\[ \delta(\delta^2 \Pi_I) + \delta(\delta^2 \Pi_{FI}) = 0 \]

Incremental   Fundamental-incremental

where

\[ \delta(\delta^2 \Pi_{FI}) = \int_\Omega \{ N_1^0 w_{1,x} \delta w_{1,x} + N_6^0 (w_{1,x} \delta w_{1,y} + w_{1,y} \delta w_{1,x}) + N_2^0 w_{1,y} \delta w_{1,y} \} \, dx \, dy \]

\[ \bar{N}^0 = \frac{\partial}{\partial x} \left( N_1^0 \frac{\partial w}{\partial x} + N_6^0 \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial y} \left( N_6^0 \frac{\partial w}{\partial x} + N_2^0 \frac{\partial w}{\partial y} \right) \]

where the stress resultants are referred to the fundamental state
THE STRESS RESULTANTS IN THE PREBUCKLING STATE ARE ASSUMED TO BE CONSTANT.

\[ N_1^0 = -\alpha_1 N_{cr}, \quad N_2^0 = -\alpha_2 N_{cr}, \quad N_6^0 = -\alpha_3 N_{cr} \]

THERMAL LOADS

\[ N_1^0 = N_{cr}^T, \quad N_2^0 = N_{cr}^T, \quad N_6^0 = 0 \]
\[ N_{cr}^T = -\frac{1}{(1-\nu)} \int_{-h/2}^{h/2} (E_{cm}f_c + E_m)(\alpha_{cm}f_c + \alpha_m)T(x, y, z) \, dz \]

- **UNIFORM TEMPERATURE RISE:**

\[ T(z) = T_f - T_i = T_{cr} \]

- **LINEAR TEMPERATURE CHANGE ACROSS THE THICKNESS:**

\[ T(z) = T_{cr}\left(\frac{z}{h} + \frac{1}{2}\right) - T_m \quad \text{WHERE:} \quad T_{cr} = T_c - T_m \]
NON-LINEAR TEMPERATURE CHANGE ACROSS THE THICKNESS: SOLVING A SIMPLE STEADY STATE HEAT TRANSFER EQUATION

\[ -\frac{d}{dz} \left( K(z) \frac{dT(z)}{dz} \right) = 0, \]

\[ T(-h/2) = T_m, \quad T(h/2) = T_c \]

SERIES SOLUTION (7 TERMS):

\[ T(z) = T_m + \frac{T_{cr}}{D} \sum_{j=0}^{5} \frac{1}{(jn+1)} \left( -\frac{K_{cm}}{K_m} \right)^j \left( \frac{z}{h} + \frac{1}{2} \right)^{(jn+1)} \]

**Series Solution Constants**:

\[ K(z) = K_{cm} f_c + K_m \]

\[ D = \sum_{j=0}^{5} \frac{1}{(jn+1)} \left( -\frac{K_{cm}}{K_m} \right)^j \]

\[ K_{cm} = K_c - K_m \]
INTERPOLATIONS

\[ u = \sum_{j=1}^{m} u_j N_j(x, y), \quad v = \sum_{j=1}^{m} v_j N_j(x, y), \quad w = \sum_{j=1}^{m} w_j N_j(x, y), \]

\[ \varphi_1 = \sum_{j=1}^{m} \varphi_j^1 N_j(x, y), \quad \varphi_2 = \sum_{j=1}^{m} \varphi_j^2 N_j(x, y) \]

\[ \psi_1 = \sum_{j=1}^{m} \psi_j^1 N_j(x, y), \quad \psi_2 = \sum_{j=1}^{m} \psi_j^2 N_j(x, y) \]
LAGRANGE POLYNOMIALS

\[ L_i^1(\xi) = \prod_{\substack{k=1 \atop k \neq i}}^{p+1} \frac{\xi - \xi_k}{\xi_i - \xi_k}, \]

\[ L_i^2(\eta) = \prod_{\substack{k=1 \atop k \neq i}}^{p+1} \frac{\eta - \eta_k}{\eta_i - \eta_k}, \quad i = 1, \ldots, p+1, \]
SHAPE FUNCTIONS

\[ N_k = L_i^1(\xi) L_j^2(\eta), \quad k = (j - 1)(p + 1) + i. \]

EIGENVALUE PROBLEM

\[ [K][\Delta] = \lambda [K_G][\Delta] \]
Comparison of the dimensionless critical load for symmetric three cross-ply laminated plates under uniaxial and biaxial compression, and shear loading (4\times4Q25 full integration)

\[ \bar{N}_{cr} = \frac{N_{cr}b^2}{E_m h^3}, \quad S = \frac{a}{h} \]

<table>
<thead>
<tr>
<th>( a/b )</th>
<th>Theory</th>
<th>Uniaxial</th>
<th>Biaxial</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FSDT [25]</td>
<td>22.3151</td>
<td>10.2024</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Present TSDT</td>
<td>22.1164</td>
<td>9.9330</td>
<td>8.7369</td>
</tr>
<tr>
<td></td>
<td>Present FSDT</td>
<td>22.3151</td>
<td>10.2024</td>
<td>8.9672</td>
</tr>
<tr>
<td></td>
<td>FSDT [25]</td>
<td>16.4340</td>
<td>3.2868</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Present TSDT</td>
<td>16.2986</td>
<td>3.2597</td>
<td>3.1285</td>
</tr>
<tr>
<td></td>
<td>Present FSDT</td>
<td>16.4340</td>
<td>3.2868</td>
<td>3.1579</td>
</tr>
</tbody>
</table>
Comparison of the uniaxial critical load for antisymmetric two cross-ply laminated square plates for various boundary conditions (2×2Q81 full integration)

<table>
<thead>
<tr>
<th>$h/a$</th>
<th>Theory</th>
<th>SSSS</th>
<th>SCSC</th>
<th>SFSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>LW3D [24]</td>
<td>11.2560</td>
<td>19.5762</td>
<td>4.7662</td>
</tr>
<tr>
<td></td>
<td>Present TSDT</td>
<td>11.5193</td>
<td>21.0224</td>
<td>4.9185</td>
</tr>
<tr>
<td></td>
<td>Present FSDT</td>
<td>11.3526</td>
<td>20.0669</td>
<td>4.8507</td>
</tr>
<tr>
<td>0.2</td>
<td>LW3D [24]</td>
<td>8.0732</td>
<td>8.9584</td>
<td>3.4867</td>
</tr>
<tr>
<td></td>
<td>TSDT [26]</td>
<td>8.769</td>
<td>11.490</td>
<td>3.905</td>
</tr>
<tr>
<td></td>
<td>FSDT [26]</td>
<td>8.277</td>
<td>9.757</td>
<td>3.682</td>
</tr>
<tr>
<td></td>
<td>Present TSDT</td>
<td>8.6514</td>
<td>10.7516</td>
<td>3.8449</td>
</tr>
<tr>
<td></td>
<td>Present FSDT</td>
<td>8.2773</td>
<td>9.7566</td>
<td>3.6817</td>
</tr>
</tbody>
</table>
\[ E_c = 380 \text{ GPa}, \quad K_c = 10.4 \text{ W/mK}, \quad \alpha_c = 7.4 \times 10^{-6} (\text{1/}^\circ \text{C}), \quad \nu_c = 0.3 \]

\[ E_m = 70 \text{ GPa}, \quad K_m = 204 \text{ W/mK}, \quad \alpha_m = 23 \times 10^{-6} (\text{1/}^\circ \text{C}), \quad \nu_m = 0.3 \]

Comparison of the critical temperature of FGM plates under nonlinear temperature rise (4×4Q25 full integration)

<table>
<thead>
<tr>
<th>n</th>
<th>Theory</th>
<th>( T_{cr} ) (a/h = 100, ( T_m = 5^\circ ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( a/b = 1 )</td>
</tr>
<tr>
<td>0.0</td>
<td>Lanhe [14]</td>
<td>24.1622</td>
</tr>
<tr>
<td></td>
<td>Present TSDT</td>
<td>24.1790</td>
</tr>
<tr>
<td></td>
<td>Present FSDT</td>
<td>24.1790</td>
</tr>
<tr>
<td>1.0</td>
<td>Lanhe [14]</td>
<td>7.6554</td>
</tr>
<tr>
<td></td>
<td>Javaheri [13]</td>
<td>7.6636</td>
</tr>
<tr>
<td></td>
<td>Present TSDT</td>
<td>7.6538</td>
</tr>
<tr>
<td></td>
<td>Present FSDT</td>
<td>7.6538</td>
</tr>
<tr>
<td></td>
<td>Present TSDT</td>
<td>4.8665</td>
</tr>
<tr>
<td></td>
<td>Present FSDT</td>
<td>4.8684</td>
</tr>
</tbody>
</table>
### NUMERICAL RESULTS (continued)

<table>
<thead>
<tr>
<th>Theory</th>
<th>$a/b = 1$</th>
<th>$a/b = 2$</th>
<th>$a/b = 3$</th>
<th>$a/b = 4$</th>
<th>$a/b = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lanhe [14]</td>
<td>3256.310</td>
<td>7640.640</td>
<td>13853.53</td>
<td>20760.85</td>
<td>27586.74</td>
</tr>
<tr>
<td>Javaheri [13]</td>
<td>3409.821</td>
<td>8539.554</td>
<td>17089.10</td>
<td>29058.47</td>
<td>44447.67</td>
</tr>
<tr>
<td>Present TSDT</td>
<td>3227.363</td>
<td>7484.611</td>
<td>13337.61</td>
<td>19674.28</td>
<td>25731.38</td>
</tr>
<tr>
<td>Present FSDT</td>
<td>3227.248</td>
<td>7483.072</td>
<td>13327.95</td>
<td>19639.08</td>
<td>25641.06</td>
</tr>
<tr>
<td>Lanhe [14]</td>
<td>1976.297</td>
<td>4691.69</td>
<td>8619.42</td>
<td>13141.43</td>
<td>17740.25</td>
</tr>
<tr>
<td>Javaheri [13]</td>
<td>2055.00</td>
<td>5157.03</td>
<td>10327.07</td>
<td>17565.13</td>
<td>26871.21</td>
</tr>
<tr>
<td>Present TSDT</td>
<td>1961.329</td>
<td>4609.313</td>
<td>8348.69</td>
<td>12530.97</td>
<td>16663.80</td>
</tr>
<tr>
<td>Present FSDT</td>
<td>1961.269</td>
<td>4608.498</td>
<td>8343.41</td>
<td>12511.06</td>
<td>16611.01</td>
</tr>
<tr>
<td>Lanhe [14]</td>
<td>1481.30</td>
<td>3478.32</td>
<td>6288.94</td>
<td>9415.58</td>
<td>12481.59</td>
</tr>
<tr>
<td>Javaheri [13]</td>
<td>1553.34</td>
<td>3899.48</td>
<td>7809.73</td>
<td>13284.08</td>
<td>20322.53</td>
</tr>
<tr>
<td>Present TSDT</td>
<td>1450.99</td>
<td>3317.92</td>
<td>5789.28</td>
<td>8352.37</td>
<td>10707.20</td>
</tr>
<tr>
<td>Present FSDT</td>
<td>1467.68</td>
<td>3404.76</td>
<td>6053.10</td>
<td>8897.18</td>
<td>11587.11</td>
</tr>
</tbody>
</table>

$T_{cr} (a/h = 10, T_m = 5^\circ)$
NUMERICAL RESULTS (continued)

MECHANICAL BUCKLING

Effect of the ratio $S$ on the critical buckling load of FGM square plates under uniaxial compression

Zirconia

\[ E_c = 151 \text{ GPa}, \quad K_c = 2.09 \text{ W/mK}, \quad \alpha_c = 10 \times 10^{-6} (1/^\circ\text{C}), \quad \nu_c = 0.3 \]
Effect of the volume fraction exponent on the critical buckling load of FGM square plates under uniaxial compression
Effect of the ratio $S$ on the critical buckling temperature of FGM square plates under uniform temperature rise (aluminium-alumina)

Effect of the volume fraction exponent on the critical buckling temperature of FGM square plates under uniform temperature rise (aluminium-alumina)
Effect of the ratio $S$ on the critical buckling temperature of FGM square plates under nonlinear temperature change across the thickness (aluminium-alumina)

Effect of the volume fraction exponent on the critical buckling temperature of FGM square plates under nonlinear temperature change across the thickness (aluminium-alumina)
CONCLUSIONS

• A displacement finite element model for stability problems is derived (shear locking is avoided).

• Comparisons of our results with other found in the literature validate the present formulation.

• The effect of the shear deformation is significant, especially for thick plates, hence it cannot be neglected. Finally, differences in the results of the TSDT and FSDT are minor but more significant for FGM plates than those of homogeneous plates (i.e., fully ceramic or fully metal plates).
I thank...

- YOU for your interest in my presentation
- Professors Glaucio Paulino, Horacio Espinosa, Fernando Rochinha, and Ney Dumont for their kind invitation
- DULCE for her help with the flight bookings, and for her excellent hospitality arrangements

That which is not given is lost
CONCLUDING REMARKS

- *Numerical simulation* continues to be a major component of computer aided engineering and manufacturing.
- Material modeling at different scales presents new challenges in developing more sophisticated and accurate computational techniques.
- A good understanding of (a) the physics and (b) the numerical method being used to simulate the process is essential for an accurate and efficient solution. Thus, engineers with good background in particular engineering subjects as well as in computational mechanics will continue to have excellent opportunities.