Numerical Analysis of Blast Induced Fracturing of Hard Rocks

Araken Lima, Celso Romanel and Deane Roehl

Civil Engineering Department
Motivation

- Rock blasting plan
- Prediction of fracture extension
- Formation of blocks
Detonation energy = stress waves + gas pressure + others
(temperature, flying rock fragments, air displacements)

Explosives: TNT – high stress waves and low gas production
    ANFO – high gas production and low energy in stress waves

shock wave propagation - μs (dynamic)
gas pressurization - ms (quasi-static)
Blast induced P-waves

- crushing around the borehole
- dense radial cracks (4-8 radii)
- dominant cracks propagate

Number of dominant cracks
Ghosh e Daemen (1995) - 8 a 12
Song e Kim (1995) - 10 a 12
2D Adaptive finite element model

- Loading: blast induced stress waves
- Rock material: homogeneous, isotropic linear elastic up to breakage
- Crushed and dense fracture zones are neglected
- Mixed mode fracture (P- and S-waves)
Fracture model

• Mixed mode I-II fracture criteria (static loading)

\[
\begin{align*}
\frac{K_{II}}{K_{IC}} + \frac{K_{II}}{K_{IIC}} &= 1 \\
\left(\frac{K_{I}}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 &= 1 \\
(\ K_I\ )^2 + (\ K_{II}\ )^2 &= (\ K_{IC}\ )^2 \\
(\ K_{I}\ )^2 + C \frac{K_{I}}{K_{IC}} \frac{K_{II}}{K_{IIC}} + \left(\frac{K_{II}}{K_{IIC}}\right)^2 &= 1
\end{align*}
\]

• Dynamic fracture criteria - mode I  Grady e Lipkin (1980)
Fracture model

• Crack propagation direction – maximum tensile tangential stress

\[ \theta_m = 2 \arctan \left[ \frac{1}{4} \left( \frac{K_I}{K_{II}} \right)^2 \pm \frac{1}{4} \sqrt{\left( \frac{K_I}{K_{II}} \right)^2 + 8} \right] \]

Mode I \( K_I = K_{IC}, K_{II} = 0 \rightarrow \theta_m = 0^\circ \)
Mode II \( K_I = 0, K_{II} = K_{IIC} \rightarrow \theta_m = -70.53^\circ \)

• Crack growth in a time interval

\[ \Delta a = c \cdot \Delta t \]

c crack growth velocity (smaller than half shear wave speed Grady & Kipp, 1979)
Finite element model

Adaptive finite element mesh

- **Mesh generation:** recursive spatial enumeration techniques
  quadtree inner domain
  boundary triangulation  (Araújo 1999)

- **Maximum element size**
  Wave reflection on element interface due to change in element size

Efficient frequency transmission: Celep e Bazant [1983] and Mullen e Belytschko [1982]: 1/10 smallest wave length

- **State variable mapping**

\[
\begin{align*}
\{t u\}^n &= \left[ N(\xi^P, \eta^P) \right] \{t-\Delta t \ q\} \\
\{t \dot{u}\}^n &= \left[ N(\xi^P, \eta^P) \right] \{t-\Delta t \ \dot{q}\} \\
\{t \ddot{u}\}^n &= \left[ N(\xi^P, \eta^P) \right] \{t-\Delta t \ \ddot{q}\}
\end{align*}
\]
• **Singular quarter point elements**  
  Henshell & Shaw (1975)  
  Barsoum (1976)

• **Finite element rosette 8 elements**

• **Stress intensity factors**

**Displacement correlation technique** (Shih, de Lorenzi and German, 1976)

**Modified crack closure method** (Raju, 1987)
Stress intensity factors

Displacement correlation technique (Shih, de Lorenzi and German, 1976)

\[
K_I = \left( \frac{\mu}{\kappa + 1} \right) \cdot \sqrt{\frac{2 \cdot \pi}{L}} \cdot (4 \cdot v_{j-1} - v_{j-2})
\]

\[
K_{II} = \left( \frac{\mu}{\kappa + 1} \right) \cdot \sqrt{\frac{2 \cdot \pi}{L}} \cdot (4 \cdot u_{j-1} - u_{j-2})
\]
Finite element model

Stress intensity factors

Modified crack closure method (Raju, 1987)

\[ G_I = -\frac{1}{2\cdot \delta a} \left\{ F_{y_i} \cdot [t_{11}(v_m - v_{m'}) + t_{12}(v_l - v_{l'})] + F_{y_j} \cdot [t_{21}(v_m - v_{m'}) + t_{22}(v_l - v_{l'})] \right\} \]

\[ G_{II} = -\frac{1}{2\cdot \delta a} \left\{ F_{x_i} \cdot [t_{11}(u_m - u_{m'}) + t_{12}(u_l - u_{l'})] + F_{x_j} \cdot [t_{21}(u_m - u_{m'}) + t_{22}(u_l - u_{l'})] \right\} \]

\[ t_{11} = 6 - 3\frac{\pi}{2} \quad t_{12} = 6\pi - 20 \quad t_{21} = \frac{1}{2} \]

\[ t_{22} = 1 \]

Normal stress distribution at crack tip

Nodal forces at singular elements
**Finite element model**

**Fracture closure control**

**Contact - penalty formulation**

Node to edge contact

gap function

\[ g(u) = \frac{A \cdot x_0 + B \cdot y_0 + C}{\sqrt{A^2 + B^2}} \]

Normal to contact forces

\[ g(u) \leq 0 \Rightarrow p = \lambda \langle -g(u) \rangle \]

Equilibrium

\[ \left[ K + K_c \right] \cdot d = R \]

with \( K_c = \sum_{j=1}^{nc} \lambda \cdot (\nabla g_j^T \cdot \nabla g_j) \)
Dynamic model

• Numerical integration of the equation of motion

\[ [M] \cdot \{\ddot{w}\} + [C] \cdot \{\dot{w}\} + [K] \cdot \{w\} = \{P(t)\} \]

Wilson θ implicit algorithm

• Pressure pulse on hole wall - Duvall (1953)

\[ p(t) = p_0 \cdot \left( e^{-\alpha t} - e^{-\beta t} \right) \]

rock constants \( \alpha \) and \( \beta \) (Aimone, 1982).
Dynamic model

Viscous dampers on the boundary - radiation condition

\[ \sigma = a \cdot \rho \cdot c_L \cdot \dot{u} \]
\[ \tau = b \cdot \rho \cdot c_T \cdot \dot{u} \]

\[ \sigma = a \cdot \rho \cdot c_L \cdot \dot{w} \]
\[ \tau = b \cdot \rho \cdot c_T \cdot \dot{u} \]

\[ M\ddot{u} + C\dot{u} + Ku = R \]
Dominant Cracks

Detonation hole with 4, 8, 10, 12 and 16 dominant cracks

Gosh/Daemem (1995) 8-12 cracks
Song/Kim (1995) 10-12 cracks

- hole size $a_0=5\text{ cm}$
- material properties: $\rho = 28\text{ Mg/m}^3$, $E = 41\text{ GPa}$, $\nu = 0,25$
  \[ K_{ID} = 1,65\text{ MPa}\cdot\text{m}^{1/2} \quad K_{IID} =1,03\text{MPa}\cdot\text{m}^{1/2} \]
- fracture propagation velocity: 1210 m/s
- $t=100\ \mu s$ hole pressure 715, 8 MPa
- finite element meshes

4 dominant fractures

16 dominant fractures
Dominant Cracks

Principal stresses

4 cracks

8 cracks
Dominant Cracks

Principal stresses

\[ \sigma_1 \]

\[ \sigma_3 \]

10 cracks

12 cracks
Dominant Cracks

Principal stresses

$\sigma_1$

$\sigma_3$

16 cracks
Examples

- 1 Blasthole with 9 dominant cracks
- 1 Blasthole with 8 dominant cracks
- 2 Blastholes with 8 dominant cracks

Granite:
- $E = 60$ GPa
- $\nu = 0.25$
- $\rho = 2.80$ Mg/m$^3$
- $K_{\text{ID}} = 1.65$ MPa.m$^{1/2}$
- $K_{\text{IID}} = 1.03$ MPa.m$^{1/2}$

Borehole:
- radius = 2.54 cm
- penalty parameter $10^7$
- penetration tol $10^3$
- initial crack length 1/3 hole radius
Example 1: Blasthole with 9 dominant cracks

Time increments $\Delta t = 20 \, \mu s$ to $50 \, \mu s$
Blasthole with 9 dominant cracks

$t = 3 \, \mu s$

4811 nodes and 2292 elements
Blasthole with 9 dominant cracks

t = 17 µs
4811 nodes and 2292 elements
Blasthole with 9 dominant cracks

$t = 37 \mu s$
4783 nodes and 2260 elements
Blasthole with 9 dominant cracks

$t = 182 \, \mu s$

6942 nodes and 3277 elements
Blasthole with 9 dominant cracks

$t = 297 \mu s$

8266 nodes and 3891 elements
Blasthole with 9 dominant cracks

$t = 330 \mu s$

7127 nodes and 3325 elements
Blasthole with 9 dominant cracks

$t = 405 \, \mu s$

7777 nodes and 3637 elements
Blasthole with 9 dominant cracks

t = 545 µs
8995 nodes and 4199 elements
Blasthole with 9 dominant cracks

\[ t = 680 \, \mu s \]

10047 nodes and 4687 elements
Blasthole with 9 dominant cracks

$t = 970 \, \mu s$

14154 nodes and 6594 elements

Experimental results on glass plate Porter (1970)
Example 2: Blasthole with 8 dominant cracks

Time increments $\Delta t = 50 \mu s$ to $125 \mu s$

![Diagram of blasthole with cracks at various angles and pressure-time graph]
Blasthole with 8 dominant cracks

t = 15 \mu s
Blasthole with 8 dominant cracks

$t = 523 \ \mu s$
(without fracture interaction control)

Stress $\sigma_1$

Horizontal displacement
Blasthole with 8 dominant cracks

\[ t = 523 \, \mu s \]

(with fracture interaction control – penalty method)

Stress \( \sigma_1 \)

Horizontal displacement
Blasthole with 8 dominant cracks

\[ t = 523 \, \mu s \]

No contact control  
Contact control
Example 3: 2 Blastholes with 8 dominant cracks

Simultaneous blasting

Blasthole 1

Blasthole 2

Crack propagation speed: 1464 m/s
2 Blastholes with 8 dominant cracks

 Crack pattern

<table>
<thead>
<tr>
<th>t = 217.7 µs</th>
<th>t = 289.7 µs</th>
</tr>
</thead>
<tbody>
<tr>
<td>free surface</td>
<td>free surface</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>
Adaptive mesh

\[ t = 538.2 \, \mu s \]

free surface
Crack pattern

$\text{t} = 538.2 \ \mu s$

free surface
Example 4: 2 Blastholes with 8 dominant cracks

Sequential blasting

Blasthole 1

- 40 cm

Blasthole 2

- 40 cm

Δt = 123.9 µs

Crack propagation speed: 1464 m/s
2 Blastholes with 8 dominant cracks

Crack pattern
\[ t = 123.9 \, \mu \text{s} \]

Sequential blasting
2 Blastholes with 8 dominant cracks

- t=199.4 µs
- t=285.4 µs
- t=311.9 µs
- t=439.9 µs
2 Blastholes with 8 dominant cracks

- $t=439.9 \mu s$
- $t=311.9 \mu s$
- $t=199.4 \mu s$
- $t=285.4 \mu s$
- $t=439.9 \mu s$

Special features
• Direction of fracture is not predetermined
• Determination of stress intensity factors (mixed mode)
• Crack closure and fracture interaction is incorporated
• Silent boundaries for simulation of stress wave radiation condition
• Efficient algorithm for adaptive mesh generation
• Quarter point quadratic element rosetts at crack tip
Further developments

• new algorithm for adaptive mesh generation
• include gas pressure on fracture walls
• automatic time step definition
• parallel processing
• fracture initiation/dominant cracks
• dynamic mixed mode criteria

→ in progress
Gas pressure on fracture walls

SIG1, t= 0.315755145 s
mesh 4096 elements, 8695 nodes